Calculations of BR$[\bar{B} \to X_s \gamma]$

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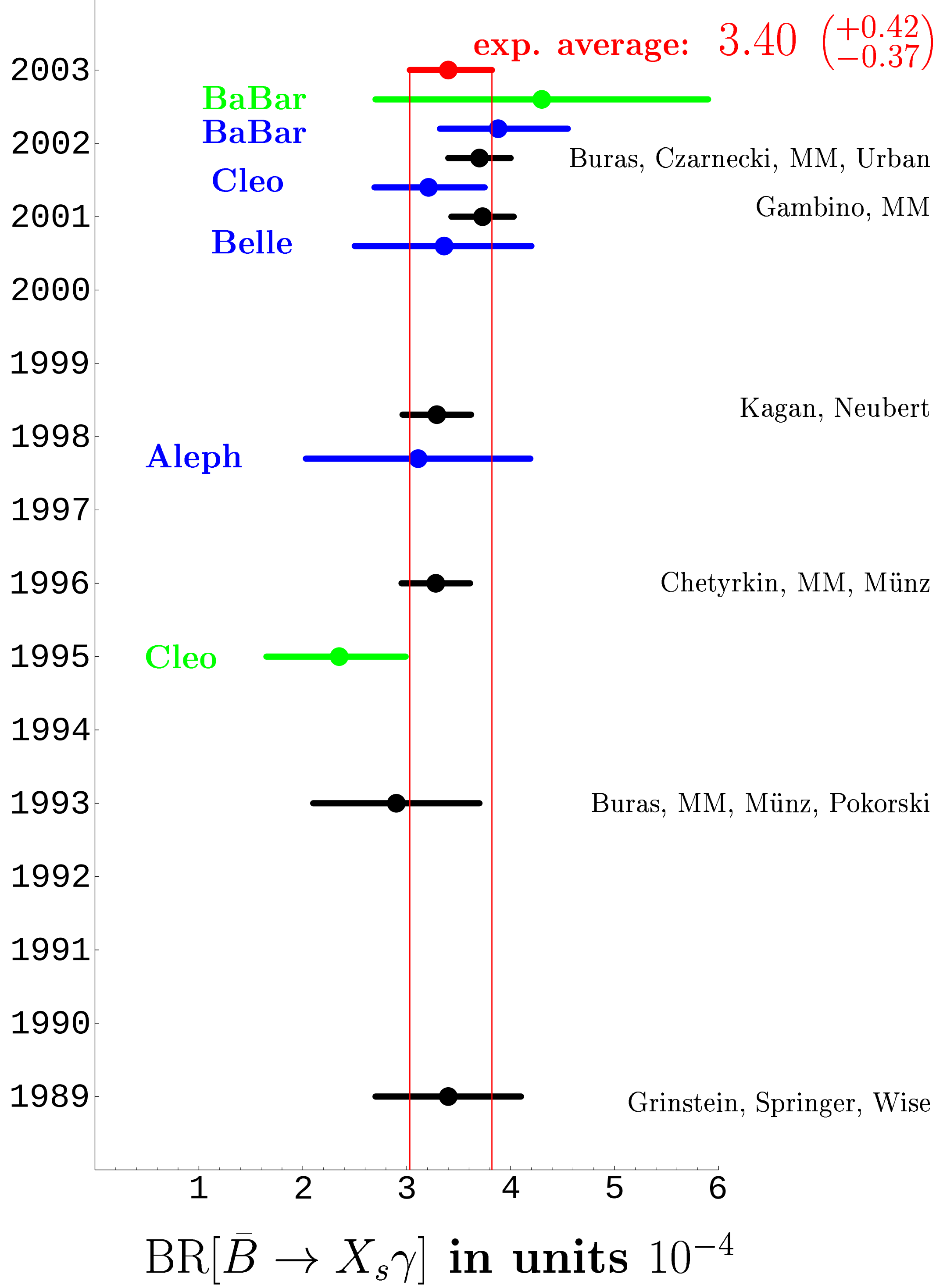
1. **SM predictions for BR$[\bar{B} \to X_s \gamma]$ vs. experiment**
2. **Perturbative calculations of $b \to X_s^{\rm parton} \gamma$**
   - (i) **completed (NLO)**
   - (ii) **future $\leftrightarrow$ charm quark loops (NNLO)**
3. **Non-perturbative effects**

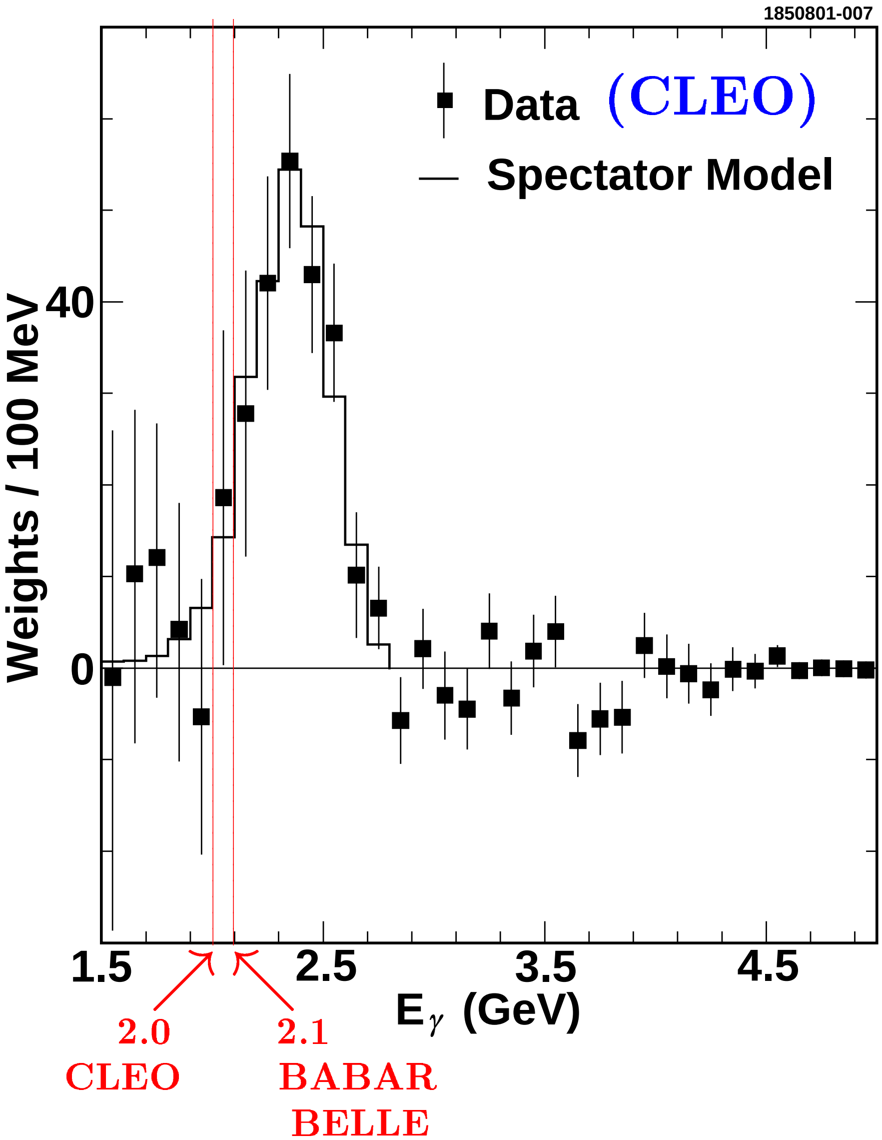
Starting point:

$$\Gamma[\bar{B} \to X_s \gamma] \simeq \Gamma[b \to X_s^{\rm parton} \gamma]$$
$$\equiv \Gamma[b \to s\gamma] \; + \; \Gamma[b \to s\gamma g] \; + \; \ldots$$

Measurements and the SM calculations
(some of them)



$\bar{B} \to X_s \gamma$ photon spectrum



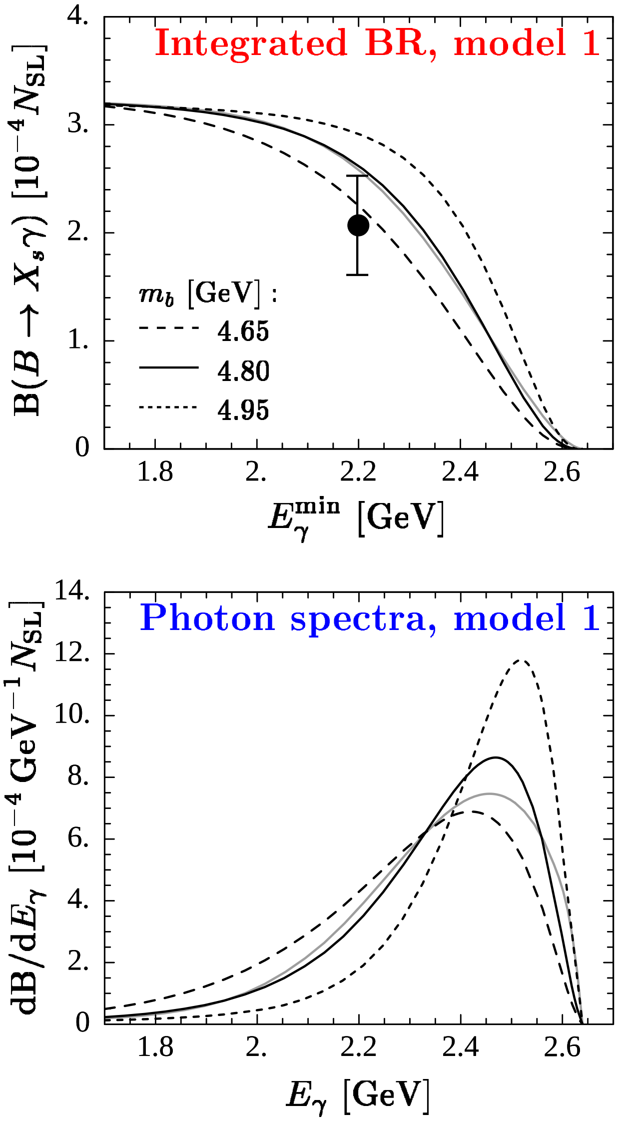
$E_\gamma > E_{\text{cutoff}}$ **to suppress** $b \to c$ **background**

Extrapolation from the experimental cutoff to lower energies $\Rightarrow$ model-dependence.

CLEO: $\sim \pm 5\%$, **BABAR:** $\sim \pm 9\%$, **BELLE:** $\sim \pm 15\%$

The measured spectrum is used for extraction of $\bar{\Lambda}$ and λ_1 parameters of HQET $\Rightarrow$ determination of V_{ub}.

Model predictions for the integrated $\bar{B} \to X_s\gamma$ branching ratio and for the corresponding photon spectra.



From: A. Kagan and M. Neubert, hep-ph/9805303, Eur. Phys. J. C7 (1999) 5.

Problems for low E_γ:

(i) Real $c\bar{c}$ bound states inside X_s ?

(ii) Photons from light-quark fragmentation ?
[A. Kapustin, Z. Ligeti and H.D. Politzer, PLB 357 (1995) 653]

Experimental average:

$$\mathrm{BR}[\bar{B} \to X_s\gamma \;\; (E_\gamma > \frac{1}{20} m_b)]_{\rm exp} \; = \; \left[3.40 \;\; \left({}^{+0.42}_{-0.37}\right)\right] \times 10^{-4}$$

After rescaling back to 1.6 GeV, one obtains

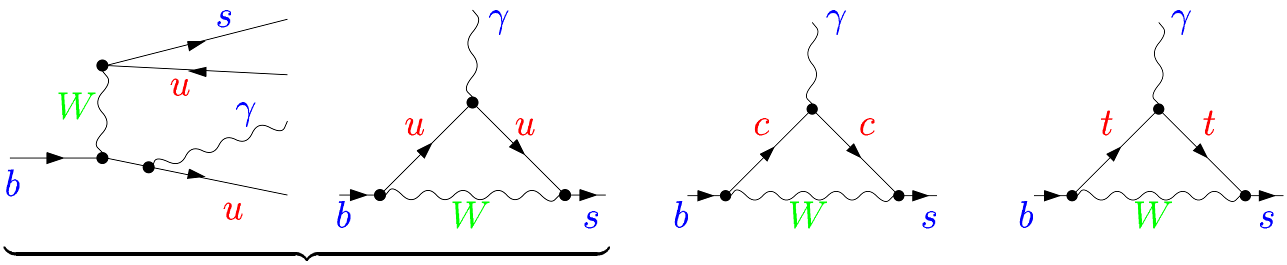
$$\mathrm{BR}[\bar{B} \to X_s\gamma \;\; (E_\gamma > 1.6 \text{ GeV})]_{\rm exp} \; = \; \left[3.28 \;\; \left({}^{+0.41}_{-0.36}\right)\right] \times 10^{-4}$$

SM prediction:

$$BR[\bar{B} \to X_s\gamma]^{\text{subtr. } \psi}_{E_\gamma > E_0} \; = \; \begin{cases} (3.57 \pm 0.30) \times 10^{-4}, & E_0 = 1.6 \text{ GeV} \\ 3.70 \times 10^{-4}, & E_0 = \frac{1}{20} m_b \end{cases}$$

The dominant uncertainty ($\sim \pm 6\%$) originates from m_c-dependence.

Electroweak transitions mediating $\bar{B} \to X_s \gamma$:



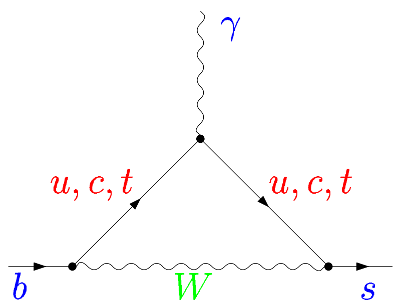
$$\left|\frac{V_{ub}V_{us}}{V_{cb}}\right| \simeq \left|\frac{V_{ub}V_{us}}{V_{ts}}\right| \simeq 2\%$$

$\sim +200\%$ $\sim -100\%$

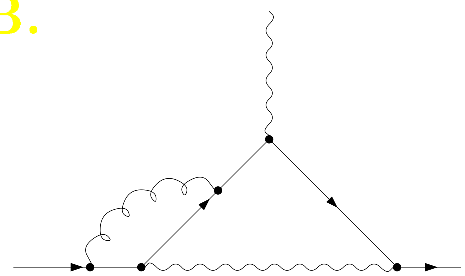
In the amplitude, after including LO QCD effects.

Examples of Feynman diagrams contributing to $b \to s\gamma$ at various orders in the renormalization-group-improved perturbation theory:

A.



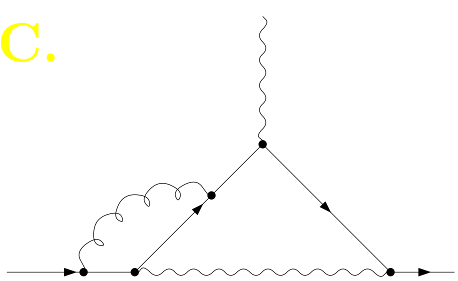
B.



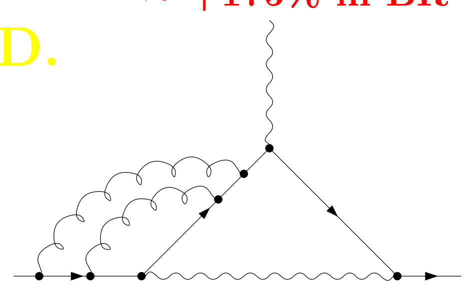
$\alpha_s \ln \frac{M_W^2}{m_b^2}$: **$\sim$ +65% in amplitude**

$\sim$ +175% in BR

C.



D.



non – logarithmic

logarithmic

$\sim$ +20% in BR

The effective Lagrangian:

$$\mathcal{L} = \mathcal{L}_{QCD\times QED}(u,d,s,c,b) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu) O_i$$

$$O_i = \begin{cases} (\bar{s}\Gamma_i c)(\bar{c}\Gamma_i' b), & i=1,2, & |C_i(m_b)| \sim 1 \\ (\bar{s}\Gamma_i b)\Sigma_q(\bar{q}\Gamma_i' q), & i=3,4,5,6, & |C_i(m_b)| < 0.07 \\ \frac{em_b}{16\pi^2}\bar{s}_L\sigma^{\mu\nu}b_R F_{\mu\nu}, & i=7, & C_7(m_b) \sim -0.3 \\ \frac{gm_b}{16\pi^2}\bar{s}_L\sigma^{\mu\nu}T^a b_R G^a_{\mu\nu}, & i=8, & C_8(m_b) \sim -0.15 \end{cases}$$

Three steps of the calculation:

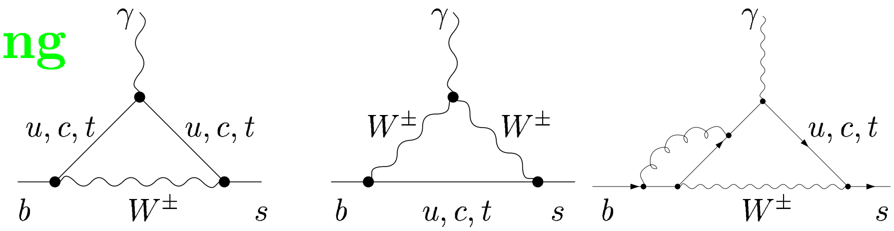
Matching: **Evaluating $C_i(\mu_0)$ at $\mu_0 \sim M_W$ by requiring equality of the SM and eff. theory Green functions.**

Mixing: **Deriving the eff. theory RGE and evolving $C_i(\mu)$ from μ_0 to $\mu_b \sim m_b$.**

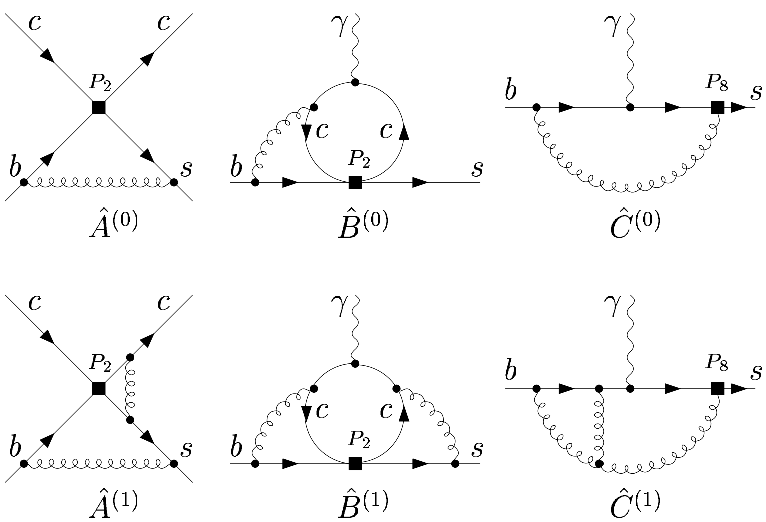
Matrix elements: **Evaluating the on-shell amplitudes at $\mu_b \sim m_b$.**

Examples of diagrams that have been calculated at LO and NLO:

Matching



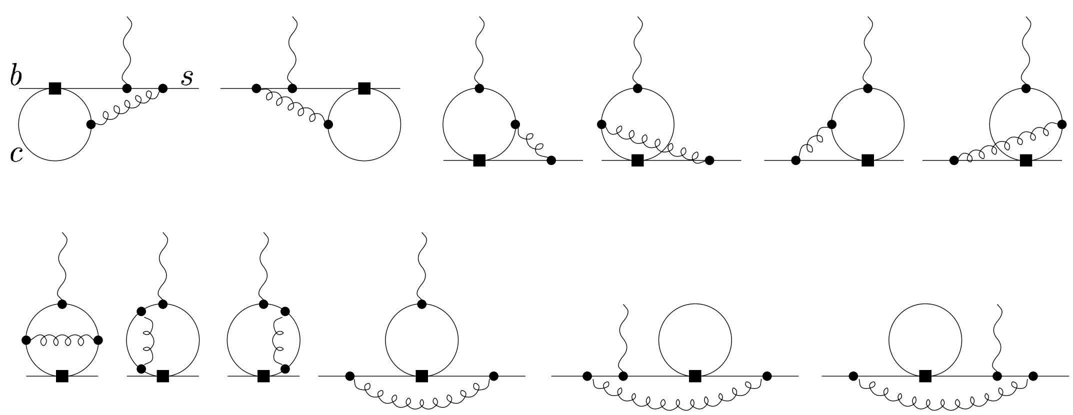
Mixing



(LO)

(NLO)

Matrix elements:



At NNLO – one gluon more in each case.

Three methods have been used for calculating 2-loop matrix elements with charm-quark loops:

1. **Mellin-Barnes transform of Feynman-parameter integrals ⇒ Expansion in m_c/m_b**
   [Greub, Hurth, Wyler, 1996]

2. **Asymptotic expansions ⇒ Expansion in m_c/m_b**
   [Buras, Czarnecki, Misiak, Urban, 2001]

3. **"Brute force" ⇒ No expansion in m_c/m_b**
   [Buras, Czarnecki, Misiak, Urban, 2002]
   Goal: Diagrams which b-quark loops
   ⇒ Formally complete NLO calculation

The $b \to s\gamma$ amplitude becomes dependent on m_c only at the NLO, via 2-loop diagrams.
The question whether we should use:

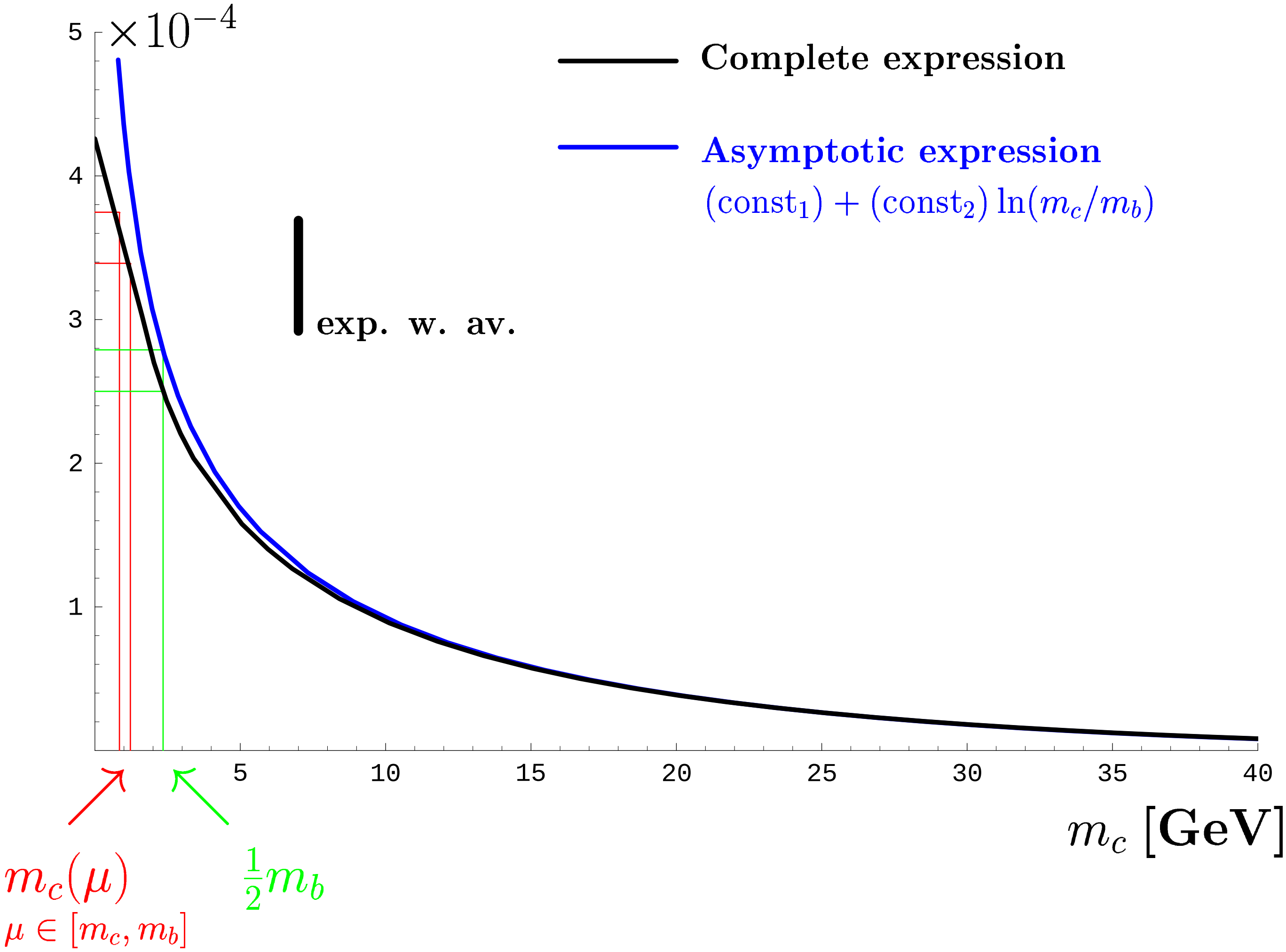
$$\frac{m_c^{\text{pole}}}{m_b^{\text{pole}}} = 0.29 \pm 0.02 \qquad \textbf{or} \qquad \frac{m_c^{\overline{\text{MS}}}(\mu)}{m_b^{\text{pole}}} = 0.22 \pm 0.04_{\;\mu \in [m_c, m_b]}$$

can, in principle, be asked only at NNLO.
However, changing m_c/m_b from 0.29 to 0.22 enhances BR$[\bar{B} \to X_s\gamma]$ by 10% ! Accuracy ↘

Way out: NNLO calculation

Charm mass dependence of

$\mathbf{BR}[\bar{B} \to X_s \gamma \quad (E_\gamma > 1.6 \text{ GeV})]$



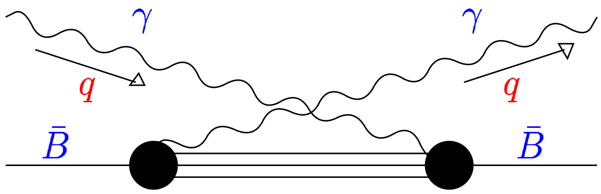
$\Rightarrow$ **Hint for the NNLO: large m_c expansion + extrapolation.**

Non-perturbative effects in $\bar{B} \to X_s \gamma$

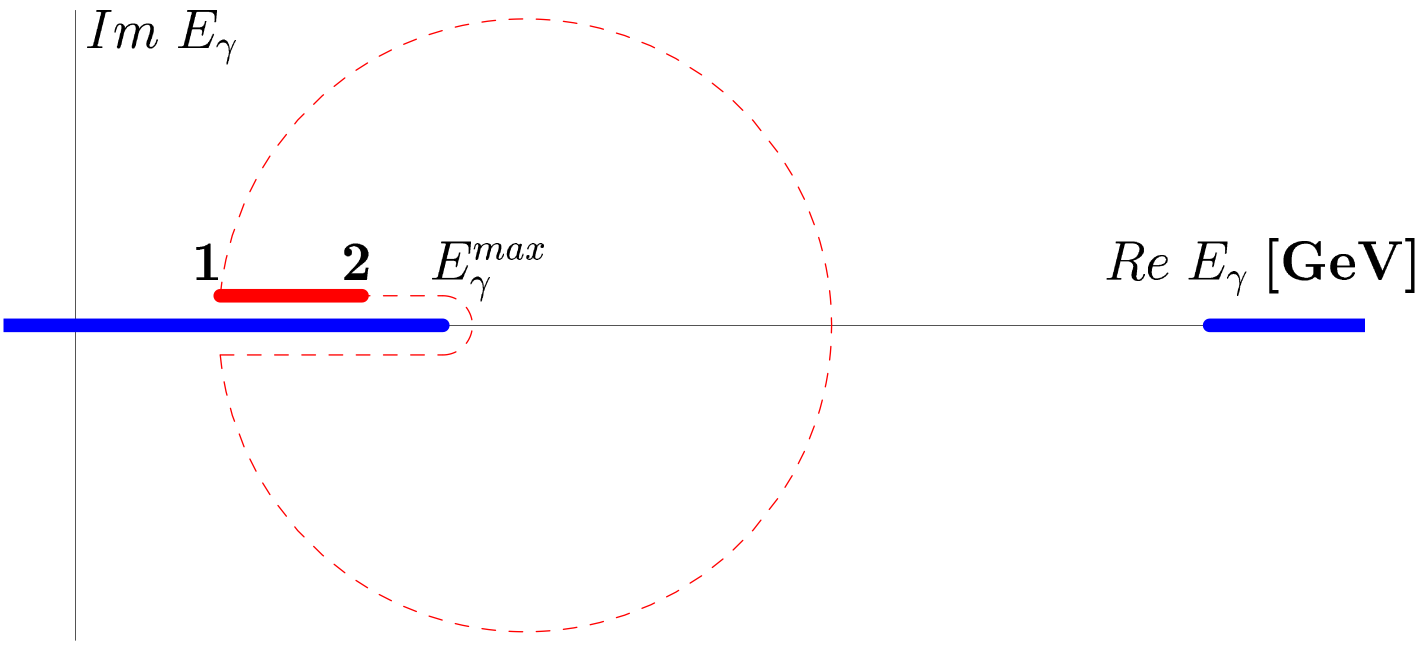
We need to sum the matrix elements of the effective Hamiltonian:

$$\Sigma_{X_s} \left| C_7 \langle X_s \gamma | O_7 | \bar{B} \rangle + C_2 \langle X_s \gamma | O_2 | \bar{B} \rangle + \ldots \right|^2$$

The "77" term in the above sum can be related via optical theorem to the imaginary part of the elastic forward scattering amplitude:

$Im\{$  $\} \equiv Im\ A$

In this amplitude, we can perform OPE when the photons are soft enough, i.e. when $|m_B - 2E_\gamma| \gg \Lambda_{QCD}$.



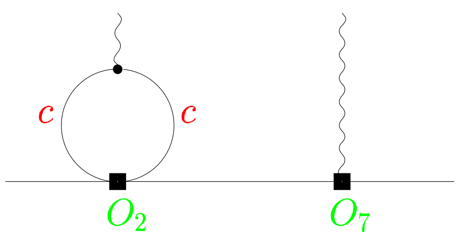
$$\int_{1\ \mathrm{GeV}}^{E_\gamma^{max}} dE_\gamma\ E_\gamma^n\ Im\ A(E_\gamma) \sim \oint_{\text{big circle}} dE_\gamma\ E_\gamma^n\ A(E_\gamma)$$

HQET gives us a double expansion:

$$\Sigma_{X_s} \mathrm{BR}[\bar{B} \to X_s \gamma]_{E_\gamma > 1 \ \mathrm{GeV}} = \left[a_{00} + a_{02} \left(\frac{\Lambda}{m_B} \right)^2 + \ldots \right]$$
$$+ \frac{\alpha_s(m_b)}{\pi} \left[a_{10} + a_{12} \left(\frac{\Lambda}{m_B} \right)^2 + \ldots \right] + \mathcal{O}\left[\left(\frac{\alpha_s(m_b)}{\pi} \right)^2 \right]$$

+ [Contributions other than the “77” term].

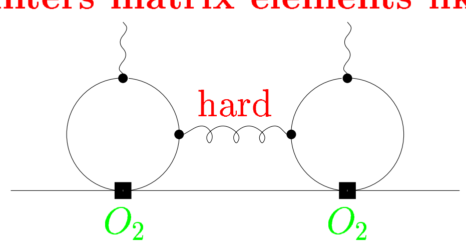
There is no OPE for the latter term. However, operators containing no charm quark are suppressed by their small Wilson coefficients. As far as the operators containing the charm quark are concerned, we know that their contribution at the leading order in α_s can be expressed as a power series:

$\langle \bar{B} |$ 

$| \bar{B} \rangle = \frac{\Lambda^2}{m_c^2} \sum_{n=0}^{\infty} b_n \left(\frac{m_b \Lambda}{m_c^2} \right)^n ,$

which can be truncated to the leading $n = 0$ term, because the coefficients b_n decrease fast with n. The calculable $n = 0$ term makes $\mathrm{BR}[\bar{B} \to X_s \gamma]$ increase by around 3%.

At $\mathcal{O}(\alpha_s)$, one encounters matrix elements like:

$\langle \bar{B} |$ 

$| \bar{B} \rangle .$

Perturbative contributions to such matrix elements are usually small, so it is enough to rely on quark-hadron duality and use the perturbative NLO expressions only. However, the intermediate ψ and ψ' contributions ($\bar{B} \to \psi^{(\prime)} X$ followed by radiative $\psi^{(\prime)}$ decay) must be subtracted on the experimental side.

Summary:

1.

$$BR[\bar{B} \to X_s\gamma]_{E_\gamma > E_0} = \begin{cases} (3.57 \pm 0.30) \times 10^{-4}, & E_0 = 1.6 \text{ GeV} \\ 3.70 \times 10^{-4}, & E_0 = \frac{1}{20} m_b \end{cases}$$

$$\text{BR}[\bar{B} \to X_s\gamma]_{\text{exp}}^{E_0 = \frac{1}{20} m_b} = \left[3.40 \ \left({}^{+0.42}_{-0.37}\right)\right] \times 10^{-4}$$

$\Rightarrow$ **Theory and experiment agree within** 1σ

2. The above agreement provides stringent constraints on new physics.

3. The dominant theoretical uncertainty is of perturbative origin. It is due to the renormalization-scheme dependence of m_c in two-loop contributions to the decay amplitude.
Calculating the NNLO corrections would remove this problem and, in consequence, reduce the theoretical uncertainty by almost a factor of 2. The main challenge at the NNLO are UV-finite parts of 3-loop massive vertex integrals with non-vanishing external momenta.

4. The numerical behaviour of $\text{BR}[\bar{B} \to X_s\gamma]$ as a function of m_c suggests that the dominant charm mass dependence originates from distances much smaller than Λ_{QCD}. In such a case, the associated non-perturbative effects would be under control.