Extracting V_{ub} Using $b \to s\gamma$

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AKL, Low, Rothstein, PRD 61, 053006 (2000)

AKL, Low, Rothstein, PLB 513, 83 (2001)

Neubert, PLB 513, 88 (2001)

Goal - Measure V_{ub}

Number of ways to measure V_{ub}:

- Exclusive measurements - $B \to \pi\ell\bar{\nu}$ and $B \to \rho\ell\bar{\nu}$
- Inclusive measurements - $b \to u\ell\bar{\nu}$ with cuts on
  - Electron energy
  - Hadronic invariant mass
  - Dilepton invariant mass
  - Mixed cuts

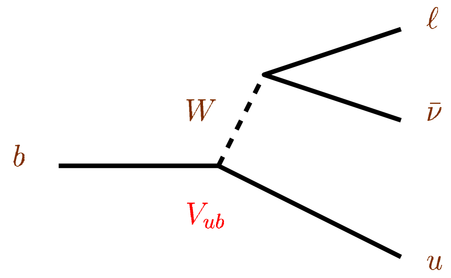
Each method has advantages and disadvantages

<u>All should be done!</u>

Inclusive electron spectrum

In principle, easy to extract V_{ub}:

Total $b \to \ell\bar{\nu}$ rate



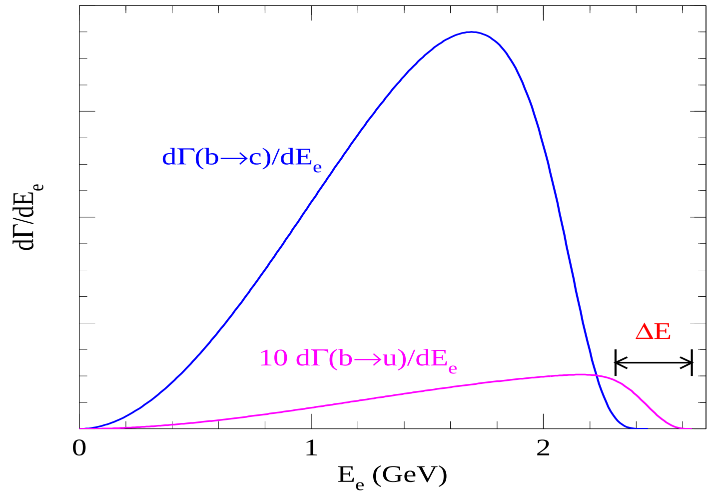
$$\Gamma = \frac{m_b^5 G_F^2}{192\pi^3}|V_{ub}|^2 \left[C_1(\alpha_s) + \frac{\lambda_1}{2m_b^2} - \frac{9\lambda_2}{2m_b^2} + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^3}{m_b^3},\ \alpha_s^3\right)\right]$$

Big problem - $b \to c$ background!

Can cut on electron spectrum

Introduces new scale ΔE

Leads to difficulties



New scale $\Rightarrow$ New problems

Scales important for $b \to u$ decay are m_b and $\Lambda_{\rm QCD}$

Calculation of total rate done in series in $\frac{\Lambda_{\rm QCD}}{m_b}$

For the cut rate, new scale ΔE: have terms like $\frac{\Lambda_{\rm QCD}}{\Delta E} \sim \mathcal{O}(1)$

Also, have series in $\alpha_s(m_b)$, including terms like

$$\alpha_s(m_b)\log^2(1-2E_e/m_b) = \alpha_s(m_b)\log^2(2\Delta E/m_b) \sim \mathcal{O}(1)$$

Both the non-perturbative and perturbative series breakdown!

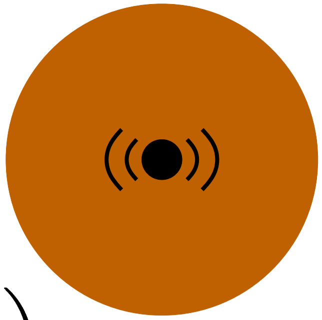
Non-perturbative series (Fermi motion)

We calculate inclusive decay using partons (b, u, etc)

This leads to partonic maximum energy $x \equiv \frac{2E_e}{m_b} \to 1$

B meson decaying, with hadronic maximum energy $x \to 1 + \bar{\Lambda}$

Difference is result of Fermi motion of b inside B



Can show from first principles that

$$\frac{d\Gamma}{dE} = \int_{2E-m_b}^{\bar{\Lambda}} dk_+ f(k_+) \frac{d\Gamma_p}{dE}(m_b^*) + \mathcal{O}\left(\frac{\Lambda}{m_b}\right)$$

with $m_b^* = m_b + k_+$

$f(k_+)$ universal $\Rightarrow$ also appears in $b \to s\gamma$

Removing $f(k_+)$

We could extract $f(k_+)$ in $b \to s\gamma$ and then use in $b \to u$

Better, skip middle step, by taking ratio of (moments of) rates

$$\left|\frac{V_{ub}}{V_{tb}V_{ts}^*}\right|^2 = \frac{3\alpha}{\pi}C_7^2(1+H_{\rm mix}^\gamma)\int_{x_B^c}^1 dx_B \frac{d\Gamma}{dx_B}\left(\int_{x_B^c}^1 du_B W(u_B)\frac{d\Gamma^\gamma}{du_B}\right)^{-1}$$

where

$$\begin{aligned} W(u_B) &= u_B^2 \int_{x_B^c}^{u_B} dx_B \Bigg\{1 - 3(1-x_B)^2 \\ &\qquad + \frac{\alpha_s}{\pi}\left[\frac{7}{2} - \frac{2\pi^2}{9} - \frac{10}{9}\log\left(1-\frac{x_B}{u_B}\right)\right]\Bigg\} \\ &\approx w_{\rm slope}(u_B - x_B^c) \end{aligned}$$

What have we done?

$$\left|\frac{V_{ub}}{V_{tb}V_{ts}^*}\right|^2 \propto \int_{x_B^c}^1 dx_B \frac{d\Gamma}{dx_B}\left(w_{\text{slope}}\int_{x_B^c}^1 du_B(u_B - x_B^c)\frac{d\Gamma^\gamma}{du_B}\right)^{-1}$$

- Formula for V_{ub} by relating $b \to u\ell\bar{\nu}$ and $b \to s\gamma$ rates
- Leading order structure function $f(k_+)$ cancels
- Accurate to $\mathcal{O}\left[\alpha_s(1-x), (1-x)^3, \Lambda/m_b\right]$
- “Simple” convolution of $b \to s\gamma$ rate with linear function

What about the breakdown of the perturbative series?

Large $\log(1 - x_B^c)$ corrections as the cut approaches 1

Stacking the logs

The logs can be summed, net effect is change in $W(u_B)$

$$W(u_B) = u_B^2 \int_{x_B^c}^{u_B} dx_B \, K\left[x_B; \frac{4}{3\pi\beta_0}\log\left(1 - \alpha_s\beta_0\ell_{x/u}\right)\right]$$

where $\ell_{x/u} = -\log(-\log(x_B/u_B))$

$W(u_B)$ is still approximately linear, just with a different slope

Correctly includes leading and next-to-leading logs

Shifts central value, but errors still dominated by Λ/m_b

What about those errors?

- Theoretical uncertainty of $\mathcal{O}\left(\alpha_s^2,\ \alpha_s(1-x_B^c),\ \Lambda/m_b\right)$
- Also have parton-hadron duality errors – difficult to quantify
- How many resonances? Only about 10% of rate
- Should also do other extractions

Quantifiable uncertainty dominated by Λ/m_b

- Subleading in Λ/m_b can have large effect in endpoint region
- Known and unknown contributions enter with new structure functions

Best solution – lower cut as much as possible

Other inclusive extractions

- Can do similar analysis for hadronic invariant mass
  - Combine rates to remove leading structure function
  - Remove large corrections by summing logs
  - Hadronic mass spectrum captures $\sim$ 80% of rate
    $\Rightarrow$ smaller duality errors?
- Dilepton invariant mass
  - Captures $\sim$ 20% of rate
  - Do not need to worry about leading structure function
  - Question as to how low can push cut
    Errors grow quickly with q^2
- Mixed cut (q^2 and m_X)
  - Best of both worlds?
  - Some model dependence

Conclusions

- V_{ub} can and should be measured in a variety of ways
- Different extractions have advantages and disadvantages
- Theoretical uncertainties on the order of 10% possible
- Best way to reduce error is to lower cuts
  (or wait for lattice)
- Must wait to see convergence of extractions