# Dinosaurs to Cavemen: 10 mins on Exclusive Semileptonic $B$ Decays 

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## Generalities

$$
\left.\begin{array}{rl}
\frac{\mathrm{d} \Gamma\left(B \rightarrow D^{*} \ell \bar{\nu}\right)}{\mathrm{d} w}= & \frac{G_{F}^{2} m_{B}^{5}}{48 \pi^{3}} r_{*}^{3}\left(1-r_{*}\right)^{2} \sqrt{w^{2}-1}(w+1)^{2} \\
& \times\left[1+\frac{4 w}{1+w} \frac{1-2 w r_{*}+r_{*}^{2}}{\left(1-r_{*}\right)^{2}}\right]\left|V_{c b}\right|^{2} \mathcal{F}_{*}^{2}(v \\
\frac{\mathrm{d} \Gamma(B \rightarrow D \ell \bar{\nu})}{\mathrm{d} w}= & \frac{G_{F}^{2} m_{B}^{5}}{48 \pi^{3}} r^{3}(1+r)^{2}\left(w^{2}-1\right)^{3 / 2}\left|V_{c b}\right|^{2} \mathcal{F}^{2}(u \\
\omega=v \cdot v^{\prime}=\left(m_{B}^{2}+m_{D^{(*)}}^{2}-q^{2}\right) /\left(2 m_{B} m_{D^{(*)}}\right) \\
r_{(*)}=m_{D^{(*)}} / m_{B}
\end{array}\right\} \begin{aligned}
& \mathcal{F}(w)=\mathcal{F}_{*}(w)=\xi_{\text {IsgurWise }}(w) \text { when } m_{Q} \rightarrow \infty \Rightarrow \\
& \mathcal{F}(1)=\mathcal{F}_{*}(1)=1
\end{aligned}
$$

## Strategy

Measure rate $\Rightarrow\left|V_{c b}\right| \mathcal{F}_{*}(w)$ at some $w$ 's
Extrapolate to $w=1$
Obtain $\left|V_{c b}\right| \mathcal{F}_{*}(1) \approx\left|V_{c b}\right|$

## Normalization

$$
\begin{aligned}
\mathcal{F}_{*}(1) & =1+c_{A}\left(\alpha_{s}\right)+\frac{0}{m_{Q}}+\frac{(\ldots)}{m_{Q}^{2}}+\ldots \\
\mathcal{F}(1) & =1+c_{V}\left(\alpha_{s}\right)+\frac{(\ldots)}{m_{Q}}+\frac{(\ldots)}{m_{Q}^{2}}+\ldots
\end{aligned}
$$

(6) perturbative $c_{A}=-0.04$ and $c_{V}=0.02$, to order $\alpha_{s}^{2}$ (Czarnecki and Melnikov)

6 Luke's theorem: $\mathcal{O}\left(\Lambda_{\mathrm{QCD}} / m_{Q}\right)$ in $\mathcal{F}_{*}(1)$ vanishes
6 Rest " (. . )" hard but small $\Rightarrow$ A. El-Khadra talk yesterday

## Extrapolation

Unavoidable? Question of money and time:
If th-error $\left(\mathcal{F}_{(*)}(1)\right)=a \Rightarrow N\left(B \rightarrow D^{*} \ell \bar{\nu}\right) \sim a^{-7 / 2}$ and $N(B \rightarrow D \ell \bar{\nu}) \sim a^{-9 / 2}$ for comparable statistical error in $\left|V_{c b}\right| \mathcal{F}_{(*)}(1)$ without assumptions about FF shapes
Use $a \approx\left(\Lambda_{\mathrm{QCD}} / m_{Q}\right)^{2}$ (or eventual lattice accuracy): $(2 \%)^{-7 / 2} \sim 10^{6}$.

Analyticity, Causality and all that nonsense (Boyd, BG, Lebed) CAVEMEN: This is solid

Constrains shape
Given a slope value (from experiment) some curvature may be unavoidable
$\Rightarrow$ straight line extrapolation may be inconsistent with QCD

## Linear vs physics-based

| Form Factor | $\rho_{\mathcal{F}}^{2}$ | $c_{\mathcal{F}}$ | $10^{2}\left\|V_{c b}\right\| \mathcal{F}(1)$ |
| :--- | :---: | :---: | :---: |
| Linear | $0.76 \pm 0.16$ | $\ldots$ | $4.05 \pm 0.45$ |
| Parabolic | $0.77_{-2.83}^{+1.18}$ | $0.01_{-3.96}^{+1.70}$ | $4.05_{-1.63}^{+1.51}$ |
| Boyd et al | $1.30 \pm 0.27$ | $1.21 \pm 0.31$ | $4.48 \pm 0.61$ |
| Caprini et al | $1.27 \pm 0.25$ | $1.18 \pm 0.26$ | $4.44 \pm 0.58$ |

$$
\mathcal{F}(w)=\mathcal{F}(1)\left[1-\rho_{\mathcal{F}}^{2}(w-1)+c_{\mathcal{F}}(w-1)^{2}+\cdots\right]
$$

CLEO, PRL82(1999)3746

## Slopes

## Fitted slope parameter

CLEO $^{a}$
$\mathrm{BELLE}^{b}$

| $B \rightarrow D^{*} \ell \bar{\nu}$, unitarity- $\rho_{A_{1}}^{2}$ | $1.67 \pm 0.11 \pm 0.22$ | $1.35 \pm 0.17 \pm 0.1$ |
| :--- | :---: | :---: |
| $B \rightarrow D^{*} \ell \bar{\nu}$, linear- $\rho_{\mathcal{F}_{*}}^{2}$ | $0.98 \pm 0.09 \pm 0.07$ | $0.89 \pm 0.09 \pm 0.0$ |
| $B \rightarrow D \ell \bar{\nu}$, unitarity- $\rho_{\mathcal{F}}^{2}$ | $1.30 \pm 0.27 \pm 0.14$ | $1.16 \pm 0.25 \pm 0.1$ |
| $B \rightarrow D \ell \bar{\nu}$, linear- $-\rho_{\mathcal{F}}^{2}$ | $0.76 \pm 0.16 \pm 0.08$ | $0.69 \pm 0.14 \pm 0.0$ |

$$
\frac{\left\langle D^{*}\left(v^{\prime}, \epsilon\right)\right| \bar{c} \gamma^{\mu} \gamma_{5} b|B(v)\rangle}{\sqrt{m_{D^{*}} m_{B}}}=h_{A_{1}}(w+1) \epsilon^{* \mu}-\left(h_{A_{2}} v^{\mu}+h_{A_{3}} v^{\mu}\right)(\epsilon
$$

${ }^{a}$ hep-ex/0007052, PRL82(1999)3746, Prv comm, K. Ecklund ${ }^{b}$ hep-ex/0111060, hep-ex/0111082, Prv comm, H. Jang

## Infamous Figure



6 HQ -symmetry $\Rightarrow \quad \rho_{\mathcal{F}}^{2}=\rho_{\mathcal{F}_{*}}^{2}$
© HQ bad? Abandon strategy? Ans: No, apples vs oranges

## Slope Differences, Theory(ZL\& BG)

$$
\begin{aligned}
\rho_{\mathcal{F}}^{2}-\rho_{\mathcal{F}_{*}}^{2}= & 0.243+0.075 \eta(1)+0.14 \eta^{\prime}(1) \\
& +1.0 \chi_{2}(1)-3.0 \chi_{3}^{\prime}(1)-0.018 \lambda_{1} / \mathrm{GeV}^{2} \simeq 0.1 \\
\rho_{A_{1}}^{2}-\rho_{\mathcal{F}_{*}}^{2}= & 0.099+0.131 \eta(1) \\
& +0.25 \chi_{2}(1)-0.007 \lambda_{1} / \mathrm{GeV}^{2} \simeq 0.17
\end{aligned}
$$

6 $\eta(w), \chi_{i}(w)$, etc, non-perturbative effects in $1 / m_{Q}$ expansion
6 Large cancellations. $\chi_{1}$ cancells altogether
© May determine some from curvature differences and FF ratios ( $R_{1}(1)$ and $R_{2}(1)$ )
© wrong direction (sign)

## Conclusions

## Use correct extrapolation

Understand slope differences
Combine $B \rightarrow D \ell \nu$ and $B \rightarrow D^{*} \ell \nu$ for better $V_{c b}$ (only after previous)

