B Physics with Lattice QCD:

Status and Prospects

Aida X. El-Khadra

(UIUC and Fermilab)

SLAC workshop on V_{xb} and V_{tx}, Dec 4-6, 2002

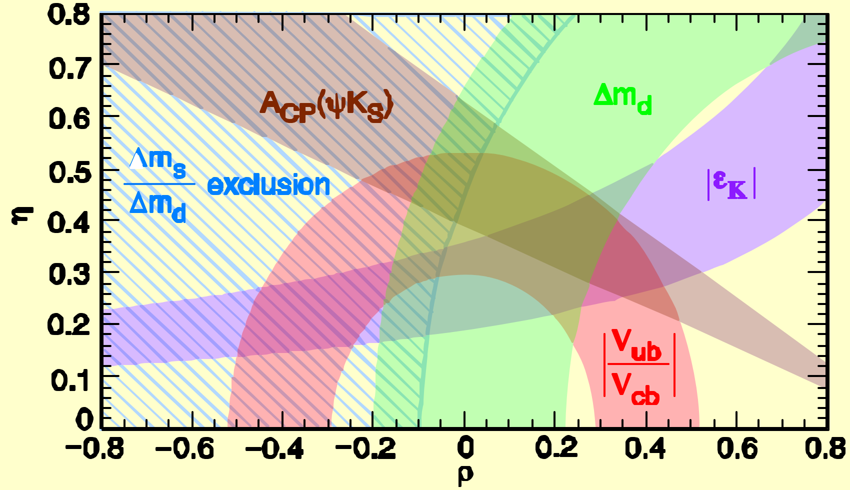
Outline:

- Motivation
- Introduction to lattice QCD
- f_B and B_B
- Semileptonic B meson decays
  - B ® $D,D^*\ lv$
  - B ® $p\ lv$
- some recent developments
- Prospects for the near future
- Conclusions

Motivation

present (2001) status:

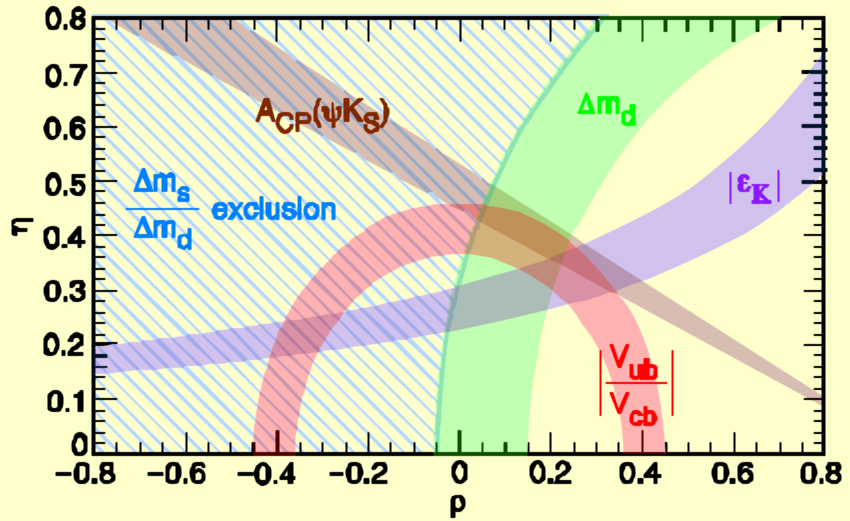
errors are dominated by theory



(CLEO-c 2001, 95% cl)

After B factories:

assuming only incremental progress for theory errors 1/2



Motivation cont'd

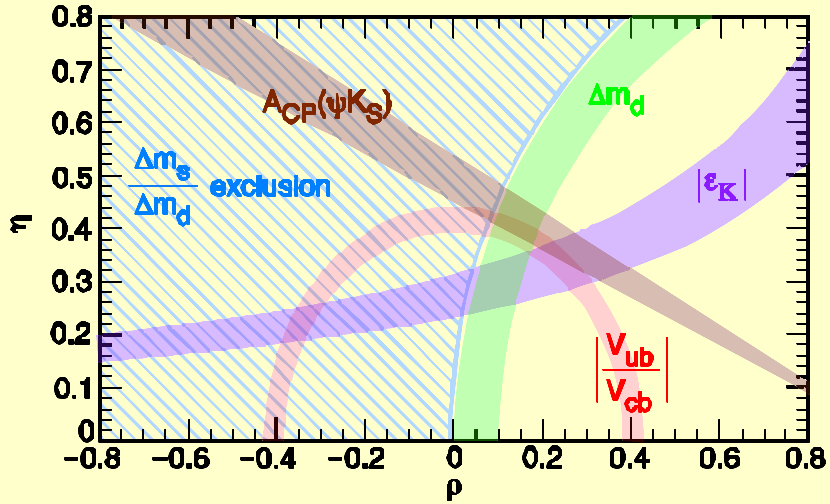
The problem:

for example $$\frac{d\Gamma(B \to \pi \ell \nu)}{dq^2} = (known) \times |V_{ub}|^2 \times |f_+(q^2)|^2$$

need the hadronic matrix elements from lattice QCD
to determine the CKM matrix elements

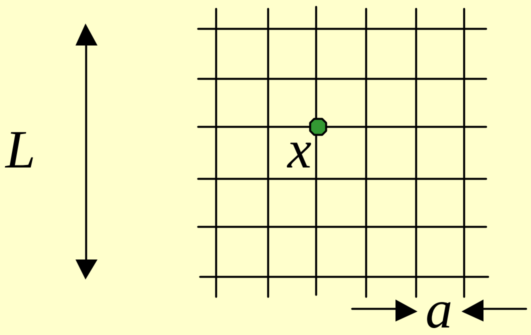
goal:

2-3% theory errors
from lattice QCD



Introduction

discretize space-time ...



fermion field lives on site: $y(x)$

gauge field lives on link: $U_m(x) = \exp[-iagA_m(x)]$

... discretize the QCD action (Wilson)

e.g. discrete derivative $\Delta_m y = \frac{1}{2a}[y(x + a\hat{m}) - y(x - a\hat{m})]$

in QCD Lagrangian $\bar{y}\partial_m y = \bar{y}\Delta_m y + a^2 c(a)\bar{y}\Delta_m^3 y + O(a^4)$

where $c(a) = c(a;a_s,m)$ depends on the QCD parameters
calculable in pert. theory

Lattice Lagrangian $\mathcal{L}^{\text{lat}} = \sum_i c_i(a; \alpha_s, m) O_i(\psi, \bar{\psi}, U_\mu)$

- in general: $\mathcal{L}^{\text{lat}} = \mathcal{L}^{\text{cont}} + O(a^n) \quad n \geq 1$

  errors scale with the typical momenta of the particles,
  e.g. $(\Lambda_{\text{QCD}}\, a)^n$ for gluons and light quarks. keep $1/a \gg \Lambda_{\text{QCD}}$

  typical lattice spacing a 0.1 fm.

- Improvement: add more terms to the action to make n large

  - gluons:

    Wilson: a^2 errors ($n = 2$)

    Lüscher + Weisz: $\alpha_s^2\, a^2$ errors

  - light quarks ($am \ll 1$):

    Wilson: a errors ($n = 1$)

    Clover (SW): $\alpha_s^2\, a$ errors, a^2 errors (Sheikholeslami+Wohlert)

    staggered: a^2 errors

    improved staggered (Asqtad): $\alpha_s\, a^2$ errors (Lepage, MILC)

Lattice Lagrangian $\mathbf{L}^{\text{lat}} = \sum_i c_i(a;a_s,m)O_i(y,\bar{y},U_m)$

•heavy quarks ($m_Q \gg \Lambda_{\text{QCD}}$ and $am_Q \not\ll 1$):

☹ SW + HQ expansion (APE, UKQCD):

start with light quark action, keep $am_Q < 1$ m_Q m_{charm}

then use HQ expansion to reach $m_Q = m_b$

corresponds to expanding c_i in m_Q: $c_i = c_i^{(0)} + am_Q\, c_i^{(1)} + \ldots$

errors: $(ap)^n$, $(p/m_Q)^n$, $(am_Q)^n$

☺ lattice NRQCD (Lepage, et al., Caswell+Lepage) :

discretize NRQCD lagrangian: valid when $am_Q > 1$

corresponds to expanding c_i in $1/m_Q$: $c_i = c_i^{(0)} + 1/(am_Q)\, c_i^{(1)} + \ldots$

errors: $(ap)^n$, $(p/m_Q)^n$

☺ Fermilab (Kronfeld, Mackenzie, AXK):

start with rel. Wilson action (+ improvement)

keep full mass dependence of c_i, add time-space asymmetry

smoothly matches heavy and light mass limits: valid for all am_Q

errors: $(ap)^n$, $(p/m_Q)^n$

systematic errors

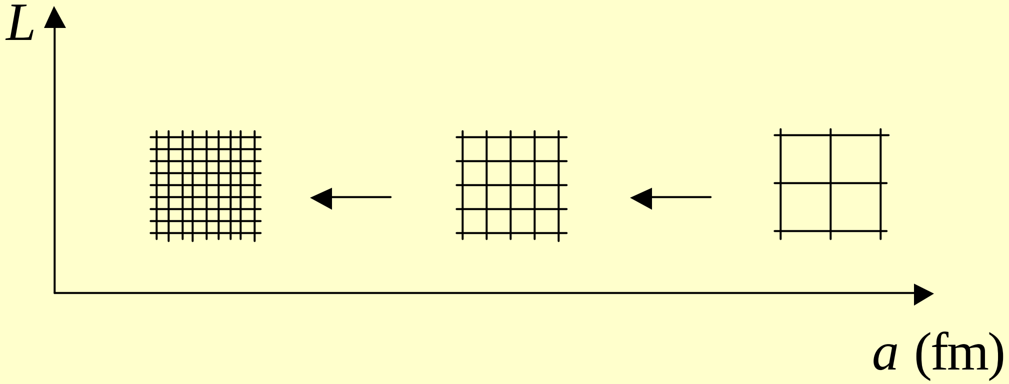
- finite lattice spacing, *a:*

$$\langle \mathbf{O} \rangle^{\text{lat}} = \langle \mathbf{O} \rangle^{\text{cont}} + O(a^n)$$

take continuum limit:

- by brute force:

  computational effort grows like ~ $(L/a)^6$



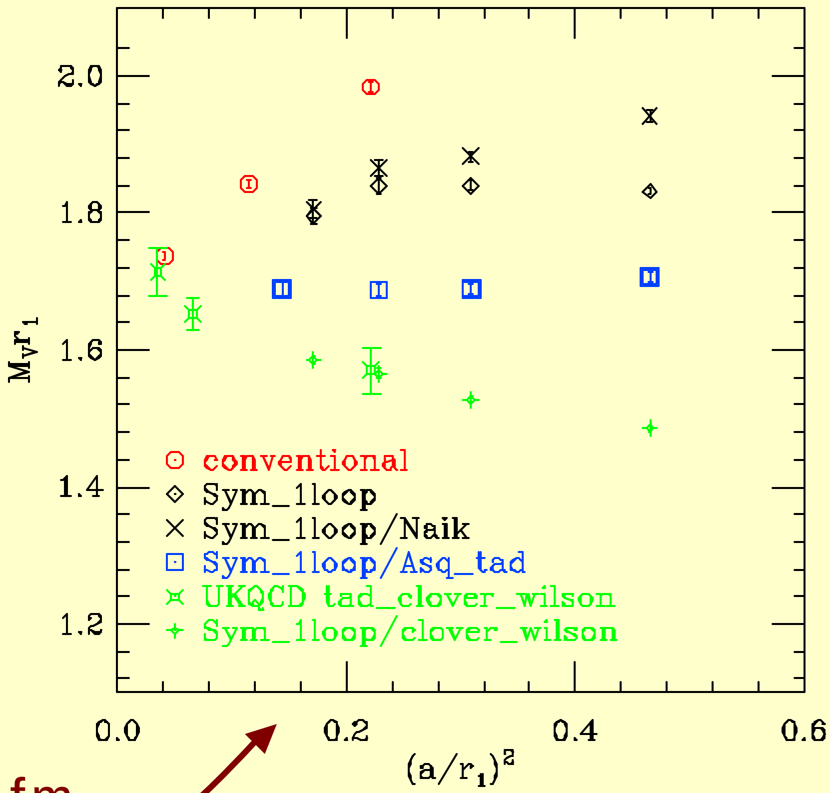
- by improving the action:

  computational effort grows much more slowly

⇨ improved actions are much better ...

MILC 1999: compare different light quark actions

example: r meson mass vs. a^2



a = 0.14 fm

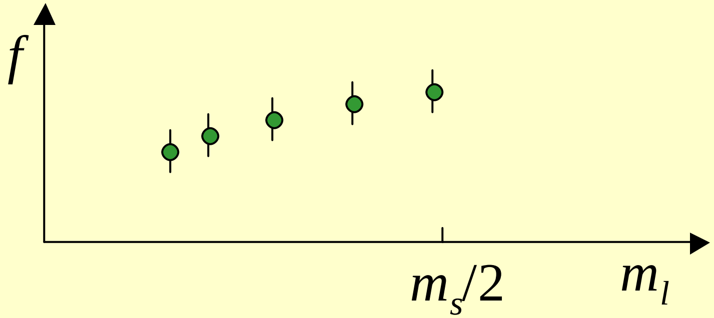
systematic errors, cont'd

- chiral extrapolation, m_l dependence:

In numerical simulations, $m_l > m_{u,d}$ because of the computational cost for small m.

use chiral perturbation theory to extrapolate to $m_{u,d}$

need $m_l < m_s/2$ and several different values for m_l (easier with staggered than Wilson-type actions)

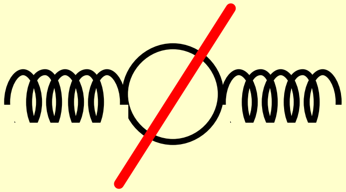


- finite Volume:

$L \sim 2$ fm okay for B's.

systematic errors, cont'd

- n_f dependence

$n_f = 0$:  quenched approximation

introduces systematic error 10 – 30 %

n_f 0: computationally difficult

keep a large, a 0.1 fm ⟹ need improved actions

so far: $n_f = 2$ with staggered and SW fermions (a^2 errors)

new MILC (2002):

$\boldsymbol{n_f = 3}$ with $m_s \neq m_{light}$ and $m_{light} = m_s/8, m_s/4 \ldots, m_s/2, \ldots, m_s$

using an improved staggered action ($a_s\, a^2$ errors)

systematic errors, cont'd

- perturbation theory:

  for example $$\left\langle J_m^{\text{cont}} \right\rangle = Z^{\text{lat}} \left\langle J_m^{\text{lat}} \right\rangle$$

  Renormalization: from $p \sim p/a$
  calculable in perturbation theory if a is small enough.
  For $a \sim 0.1$ fm, $a_s \sim 0.25$:
  with 1-loop pert. thy, errors ~ $O(\alpha_s^2) \sim 5\%$

  Need 2-loop lattice perturbation theory for ~ few% errors
  difficult, especially with improved actions!

  use:

  - automated perturbation theory (Lüscher+Weisz, Lepage, et al)
  - computational methods (di Renzo, et al)
  - numerical methods (Lepage, Mackenzie, Trottier, ...)
  - nonperturbative methods (Alpha, APE, ...)

  example: static self energy to 3-loops (Trottier, et al, di Renzo+Scorzato)
  a_s to 2-loops (in progress)

What are the “easy” lattice calculations ?

For stable (or almost stable) hadrons, masses and amplitudes with no more than one initial (final) state hadron, for example:

- $p, K, D, D^*, D_s, D_s^*, B, B^*, B_s, B_s^*$ mesons
  masses, decay constants, weak matrix elements for mixing, semileptonic, and rare decays

- charmonium and bottomonium (η_c, J/y, h_c, …, h_b, U(1S), U(2S), ..) states below open D/B threshold
  masses, leptonic widths, electromagnetic matrix elements

This list includes most of the important quantities for CKM physics. Excluded are r mesons and other resonances.

"easy" quantities for most CKM elements ...

V_{ud}	$\mathbf{V_{us}}$ $K \to p\ ln$	$\mathbf{V_{ub}}$ $B \to p\ \ ln$
$\mathbf{V_{cd}}$ $D \to p\ ln$ $D \to ln$	$\mathbf{V_{cs}}$ $D \to K\ ln$ $D_s \to\ ln$	$\mathbf{V_{cb}}$ $B \to D, D^*\ ln$
$\mathbf{V_{td}}$ $B^0 - \overline{B^0}$ mixing $K^0 - \overline{K^0}$	$\mathbf{V_{ts}}$ $B_s - \overline{B_s}$	V_{tb}

f_B and B_B

- $n_f = 0$: $f_B = 173\,(23)$ MeV (Yamada average at Lattice 2002)

  has been stable in the last four years

  dominant error: n_f dependence

- $n_f \quad 0$: most results have $n_f = 2$

  "heavy" (valence) light quarks with $m_l \quad m_s/2$

  $f_B\,(n_f \quad 0)\,/\,f_B\,(n_f = 0) = 1.1 - 1.2$

  $f_{Bs}\,/\,f_{Bd} = 1.16\,(5)$ agrees with $n_f = 0$

- new in 2002:

  include chiral logs in chiral extrapolation

  increases $f_{Bs}\,/\,f_{Bd} \rightarrow 1.3$

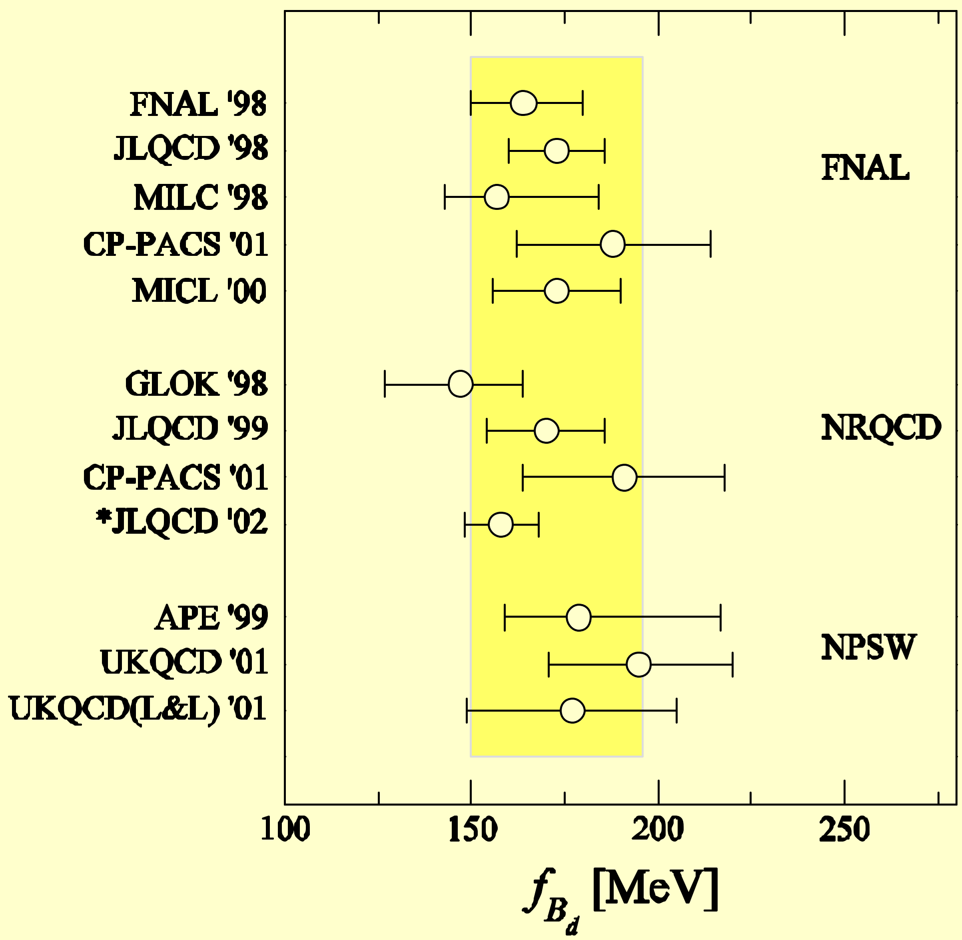
  increases the systematic error due to m_l dependence

  1st result with $n_f = 3$ (MILC, Lattice 2002)

  but also with "heavy" valence light quarks

Yamada review at Lattice 2002

f_B quenched



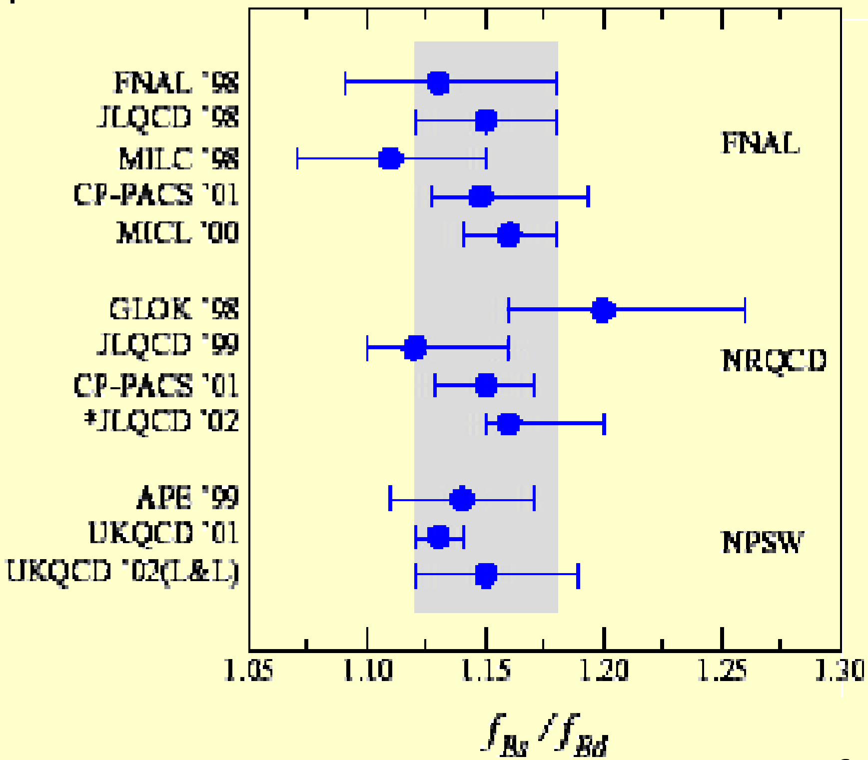
f_B = 173 (23) MeV

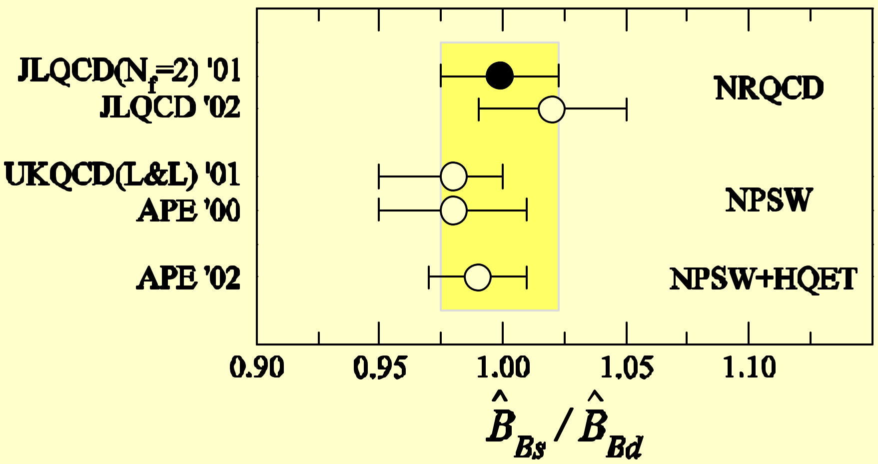
f_{Bs} = 200 (20) MeV

f_D = 203 (14) MeV

f_{Ds} = 230 (14) MeV

Yamada Lattice 2002 review: $(n_f = 0)$





$$f_{Bs} / f_{Bd} = 1.15\ (3)$$

$$B_{Bs} / B_{Bd} = 1.00\ (3)$$

$$= 1.15\ (5)$$

m_l dependence: chiral logarithms

- When $m_l > m_{u,d}$, use ChPT to extrapolate $f_p(m_l)$ to the physical u,d quark masses (Gasser+Leutwyler, Sharpe, Bernard, Golterman, Shoreish):

$$f_p(m_l) = f(1 + a x_l + b x_l \log x_l + c x_l^2 \ldots)$$

where $x_l = 2 B_0 m_l / (4p f)^2 \ll 1$, and b is known.

chiral log becomes important for small m_l need $m_l < m_s/2$

- for f_B (Grinstein, et al, Booth, Sharpe, Zhang):

$$f_B(m_l) = f(1 + a x_l + b(1 + 3g^2) x_l \log x_l \ldots)$$

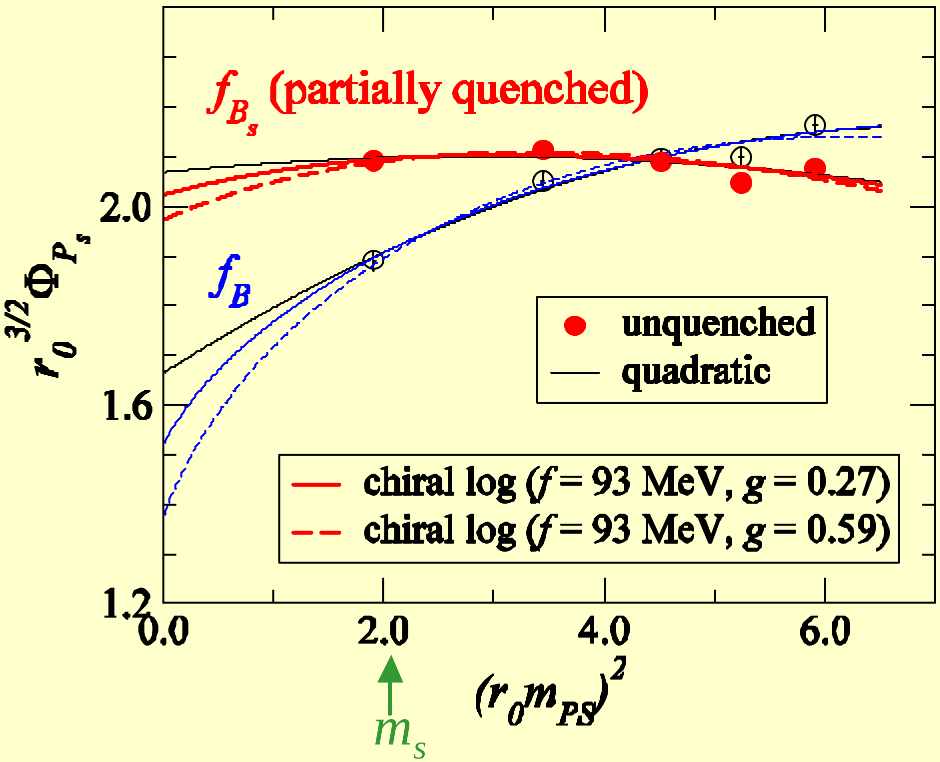
where $g \sim g_{B^*Bp}$ is the B^*Bp coupling, which is poorly known.

from the D^* width (CLEO) $g^2 \sim 0.35$.

- the problem:
(valence) $m_l > m_s/2$ in most simulations to date

chiral logarithms cont'd

Hashimoto (Lattice 2002)



$f_{Bs} / f_{Bd} = 1.24 - 1.38$

agrees with
Kronfeld + Ryan (2002)
analysis of chiral logs.

- Kronfeld+Ryan: chiral logs are small for B_B
- Becirevic, et al: use double ratios: $(f_{Bs} / f_{Bd}) / (f_K / f_p)$

solution: simulations with $m_l < m_s/2$

Semileptonic B meson decays

$B \to D, D^{*} l\nu$:

e.g. $$\frac{d\Gamma(B \to D^{*} l\nu)}{dw} = (\text{known}) \quad (w^2 - 1)^{1/2} \quad |V_{cb}|^2 \; |\mathcal{F}_{B \to D^*}(w)|^2$$

- calculate $\mathcal{F}(1)$ in lattice QCD from double ratios,
  e.g.
  $$R_+ = \frac{\langle D | V_0 | \overline{B} \rangle \langle \overline{B} | V_0 | D \rangle}{\langle D | V_0 | D \rangle \langle \overline{B} | V_0 | \overline{B} \rangle} = | h_+(1) |^2$$
  - when $m_b = m_c$: $R \equiv 1$.
  - systematic errors scale with $R - 1$, not R.
  - with $O(a)$ $O(1/m)$ improved action R is correct through $1/m^2$

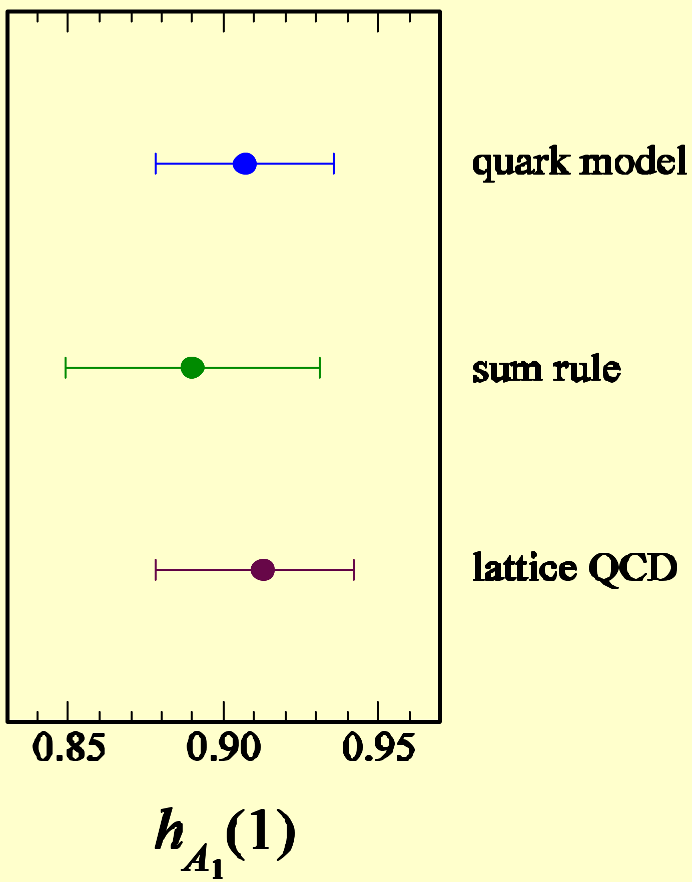
(Kronfeld)

$B \to D, D^* l\nu$ cont'd

FNAL (2001):

$$\mathcal{F}_{B\to D^*}(1) = 0.913^{+0.024}_{-0.017} \pm 0.016^{+0.003+0.000+0.006}_{-0.014-0.016-0.014}$$

stat. | pert. thy | a | m_l | n_f



- lattice result is obtained at $n_f = 0$
- $n_f = 3$ will be done soon
- chiral extrapolation error will be reduced by having $m_l < m_s/2$

B*® *p ln

- $p_\pi\,(q^2)$ dependence: $\vec{p}_p \neq 0$

$$\langle p \,|V_m|\, B\rangle^{\mathrm{lat}} = \langle p \,|V_m|\, B\rangle^{\mathrm{cont}} + O(ap_p)^n$$

$p_p \lesssim 1\mathrm{GeV}$

improved actions help (keep n large)

- experiment: measure dG/dp_π for $0 \ll p_p < m_B/2$

× old solution:

extrapolate to $p_p < m_B/2$ $(q^2 = 0)$ by assuming shape (pole dominance) — introduces model dependence!

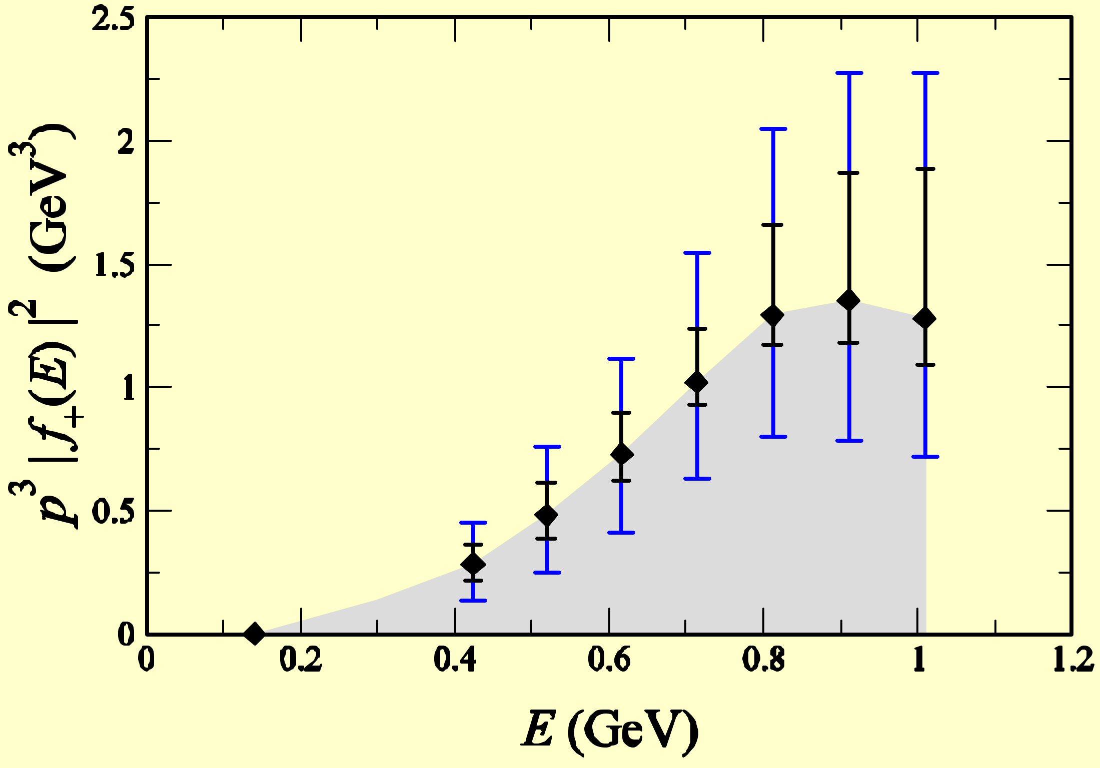
✓better solution:

limit recoil momentum range, e.g.

$$400\ \mathrm{MeV} < p_\pi < 800\ \mathrm{MeV}$$

Note: okay for D decays

FNAL (2000): partial differential decay rate



B® *p* ***l****n* cont'd

✓ even better solution:

moving NRQCD (Foley, Lepage, 2002):

give the B meson momentum p_B

write the b quark momentum as

$$p_b^m = m_b\, u^m + k^m$$

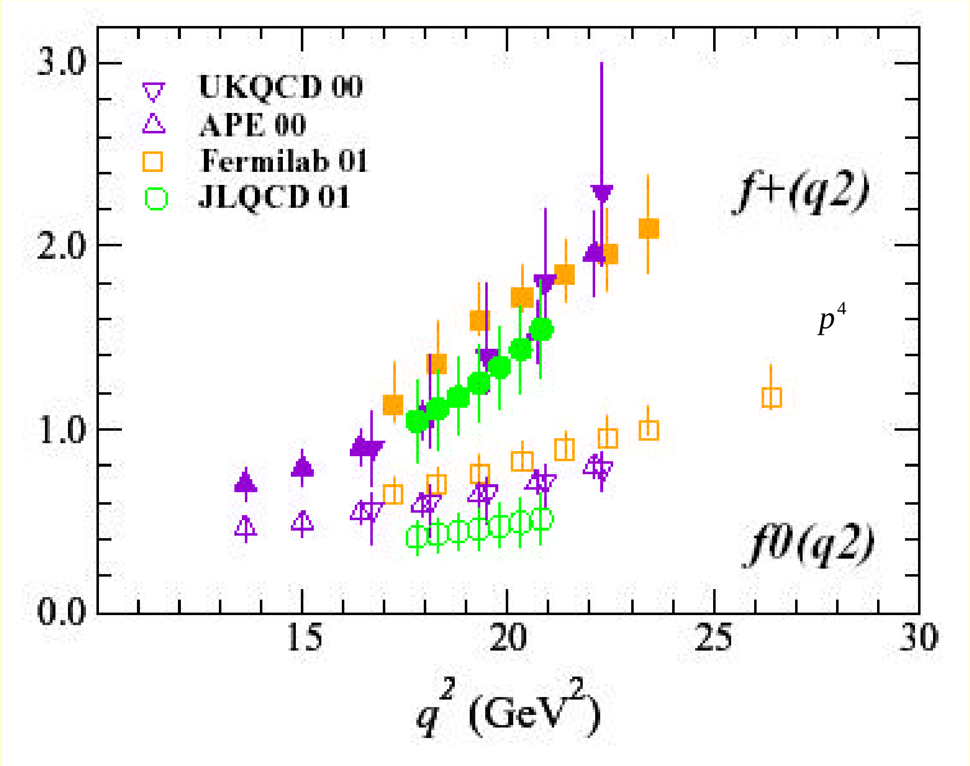
remove $m_b\, u^m$ from the dynamics
reduces to regular NRQCD for b quark at rest.

no numerical simulations with this formalism yet

This idea can also be adapted to the Fermilab approach using HQET.

B® p ln cont'd

to date: $n_f = 0$ only



p^4

errors are large,
e.g. FNAL:

$$T_B \equiv \int_{0.4\text{GeV}}^{0.8\text{GeV}} dp \, \frac{p^4 \,|\, f_+(E) \,|^2}{E}$$

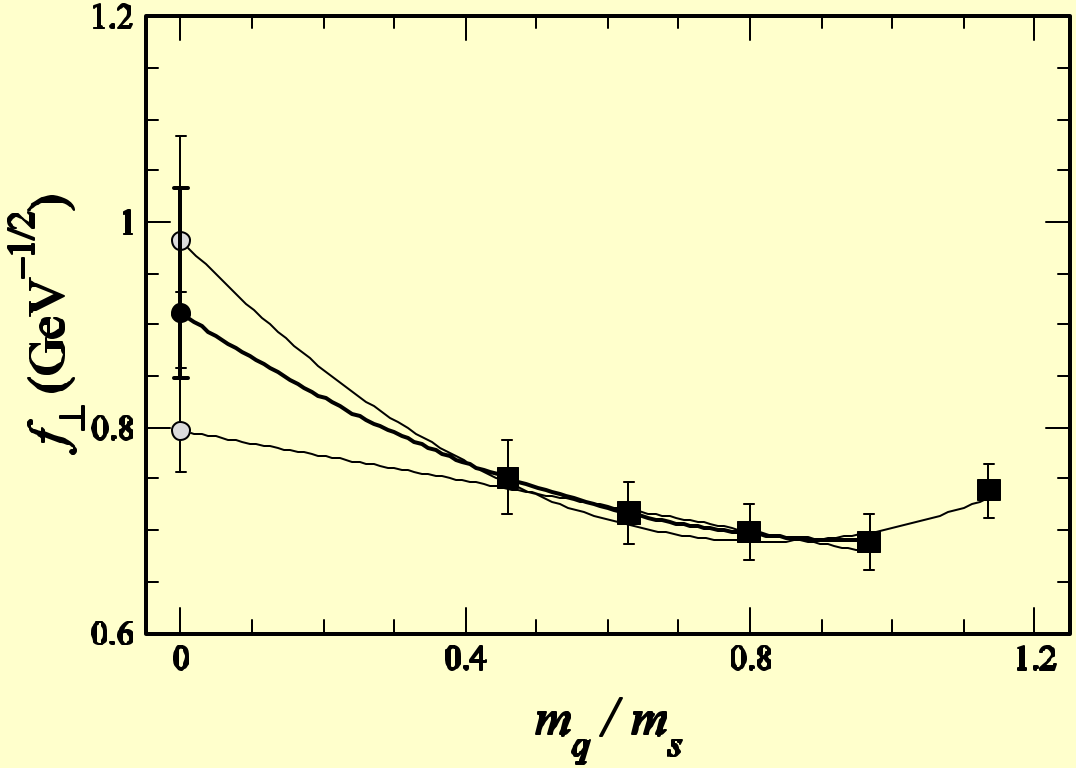
$$= \left(0.294 \begin{matrix} +0.063 \\ -0.031 \end{matrix} \begin{matrix} +0.076 \\ -0.082 \end{matrix} \right) \text{GEV}^4$$

stat sys

also: new calculation on anisotropic lattices (Shigemitsu, et al)

$B \to \pi \ell \nu$ cont'd

FNAL (2000): chiral extrapolation



dominant systematic error ~ 15 – 20%

Some Recent Developments

- ✓ highly improved actions:

light quarks:

improved staggered action: correct through ~ $O(a^2)$
computationally affordable $\boldsymbol{n_f = 3}$ feasible!

MILC (2002): $\boldsymbol{n_f = 3}$ configurations at $a = 0.12$ fm
with $m_s \neq m_{light}$ and $m_{light} = m_s/8, m_s/4 \ldots, m_s/2, \ldots, m_s$

heavy quarks:

NRQCD: correct through ~ $O(a^2)$, $O(v^4)$
$O(v^6)$ in progress

Fermilab: correct through ~ $O(a_s a)$, $O(p/m)$
$O(a^2)$, $O(p^2/m^2)$ in progress

- ✓ automated perturbation theory

get pert. results for new actions quickly
2-loop calculations in progress

Recent Developments cont'd

- the new MILC configurations include realistic sea quark effects.

  strategy:

  the only free parameters in lattice QCD lagrangian:
  quark masses and a_s

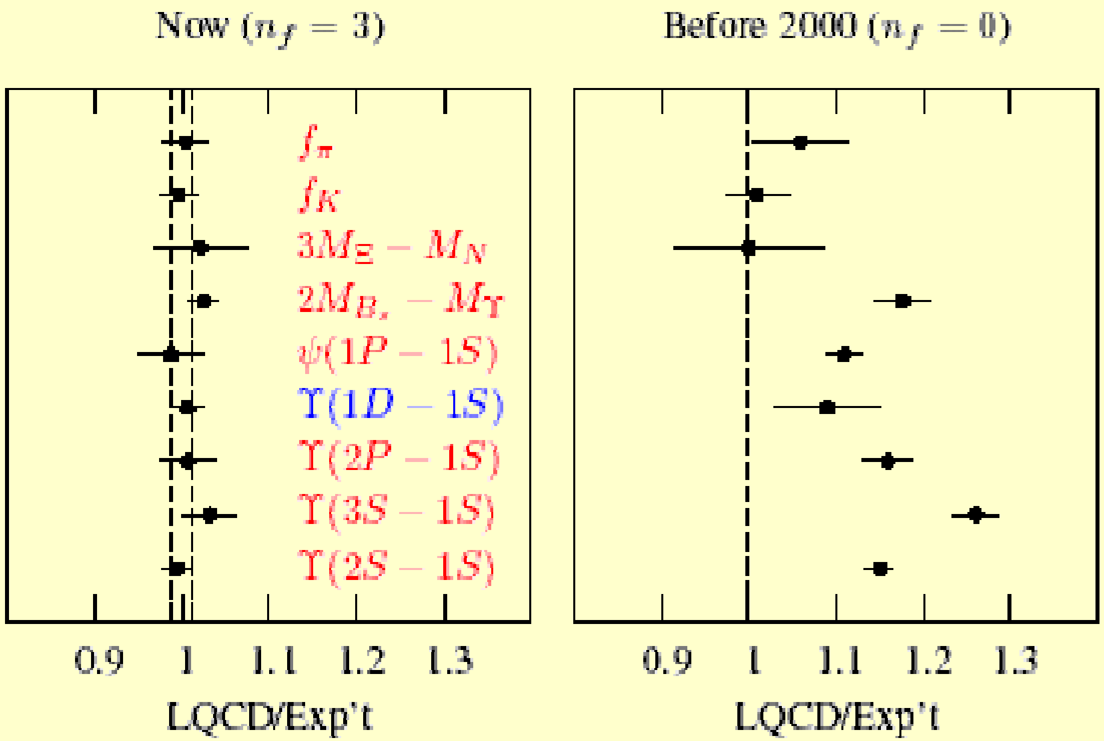
  tune the lattice QCD parameters using experiment:
  $m_{u,d}$, m_s , m_c , m_b using p, K, J/y , U meson masses
  a_s using 1P-1S splitting in U system

  all other quantities should agree with experiment …
  try this for some easy quantities …

Recent Developments cont'd

HPQCD + MILC (preliminary)

lattice QCD/experiment



works quite well!

Prospects for the near future

work currently in progress using the MILC configurations
within the next year we can expect first results for …

- ✓ U and J/y systems using NRQCD and Fermilab actions
  - test the new heavy quark actions
    - a_s, m_b, m_c
- ✓ p, K meson systems
  - using improved staggered light quarks with $m_l < m_s/2$
  - masses, decay constants, mixing, SL form factors

Prospects for the near future cont'd

... and for ...

✓ D, D_s, B, B_s meson systems

using improved staggered light quarks with $m_l < m_s/2$

masses (splittings), decay constants, mixing, SL form factors

comparison with CLEO-c essential to test lattice results

expect initial accuracy of < 10% errors
with an ultimate goal of 2-3% errors.

Conclusions

- ✓ lattice QCD calculations are an important component of the physics program of the B factories.

- ✓ current status:
  - f_B, B_B, $f_{B\to\pi}(E)$ to 10-30% accuracy
  - $\mathcal{F}_{B\to D^*}(1)$ to few % accuracy

- ✓ lattice results with realistic sea quark effects are here!
  - expect to see a growing number of results within the next year

- ✓ made possible with improved staggered action

- ✓ improved heavy quark actions NRQCD/Fermilab

- ➢ 2-3% accuracy requires 2-loop pert. matching
  - need to redo pert. calculations for the new actions
  - automated pert. theory methods help

- ➢ CLEO-c experiment is important for testing lattice QCD