Getting rid of the Fermi motion by choosing the proper kinematical variable

U. Aglietti, M.C., P. Gambino, NPB637(2002)427 [hep-ph/0204140]

$$\text{Generic process: } H_Q \rightarrow X + Y_{\text{non-}QCD}$$

H_Q: hadron containing a heavy quark Q with $m_Q >> \Lambda_{QCD}$

X: inclusive light-quark state (X_s, X_u)

$Y_{\text{non-}QCD}$: non-coloured particles (γ, $l\nu$, …)

HQ "Fermi motion" : $\Lambda^2_{QCD} \ll m^2_X \sim \Lambda_{QCD} E_X \ll E^2_X$

- ➢ *Universal shape function*, independent of the final state

Shape function from *perturbative* QCD

From the resummation theory in perturbative QCD:

$$\frac{d\Gamma}{dx} = \Gamma^0 \left[K(\alpha_s) f(x) + D(x;\alpha_s) \right]$$

Coefficient function
short distance,
process dependent

QCD shape function
IR divergent,
process independent

Remainder function
short distance,
process dependent

If the end-point singularity is $x = 1$, $f(x)$ resums all IR logs of the form:

$$\alpha_s^k(E_X)\left(\frac{\ln^n(1-x)}{1-x}\right)_+ \qquad 0 \leqslant n \leqslant 2k-1$$

QCD vs HQET shape functions

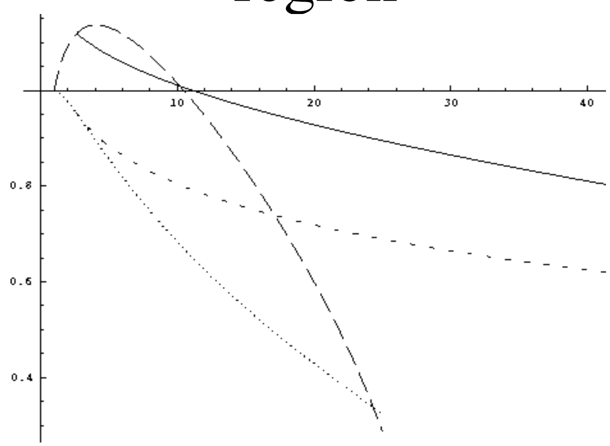
$$f(x, m_B) = c(x, m_B, \mu) \otimes f^{\mathrm{HQET}}(x, \mu)$$

includes:

- soft gluons up to scales $\sim m_B$
- hard collinear gluons

defined in terms of hadronic *physical* quantities only!

Resummed coefficient function, short-distance in the shape-function region



U. Aglietti, PLB515(2001)308

includes:

- soft gluons up to scales $\sim \mu < m_B$

usual definition of the shape function:

$$f^{\mathrm{HQET}}(k_+) = \frac{\langle B|\bar{h}_v \, \delta(k_+ - iD_+) \, h_v|B\rangle}{\langle B|\bar{h}_v h_v|B\rangle}$$

1. photon spectrum in $B \to X_s \gamma$

kinematics $\quad M_X \ll m_B \Leftrightarrow M_X \ll E_X$

$$E_\gamma = \frac{m_B}{2}\left(1 - \frac{M_X^2}{m_B^2}\right), \quad E_X = \frac{m_B}{2}\left(1 + \frac{M_X^2}{m_B^2}\right) \to \frac{m_B}{2}$$

$$\frac{d\Gamma_{rd}}{dx} = \left| V_{tb} V_{ts}^* \right|^2 \Gamma_{rd}^0 \left[K_{rd}(\alpha_s) f(x) + D_{rd}(x;\alpha_s) \right]$$

U. Aglietti, NPB610(2001)293

$$x \equiv \frac{2E_\gamma}{m_B} = \frac{2t}{1+t} \quad \text{with} \quad t \equiv \sqrt{1 - \frac{M_X^2}{E_X^2}}$$

$$\Gamma_{rd}^0 \equiv \frac{\alpha_e}{\pi} \frac{G_F^2 m_b^3 \overline{m}_b^2}{32\pi^3}$$

2. triple-differential distribution in $B \to X_u l \nu$

- 3 scales: m_B , E_X , m_X (in this case $2E_X \neq m_B$)
- *good* kinematical variables: (the *relevant* hard scale is E_X , not m_B)

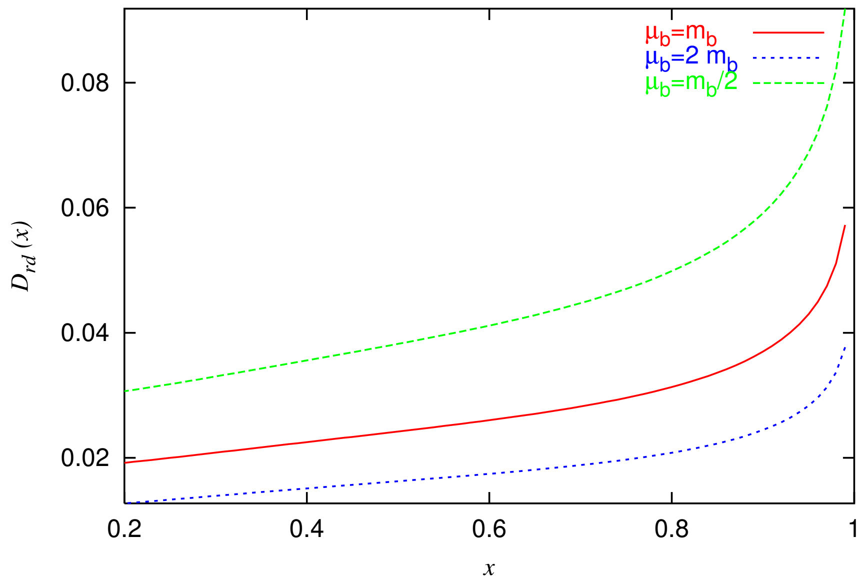
$$w = \frac{2E_X}{m_B}, \quad x_l = \frac{2E_l}{m_B}, \quad \boxed{\xi = \frac{2t}{1+t}} \qquad t \equiv \sqrt{1 - \frac{M_X^2}{E_X^2}}$$

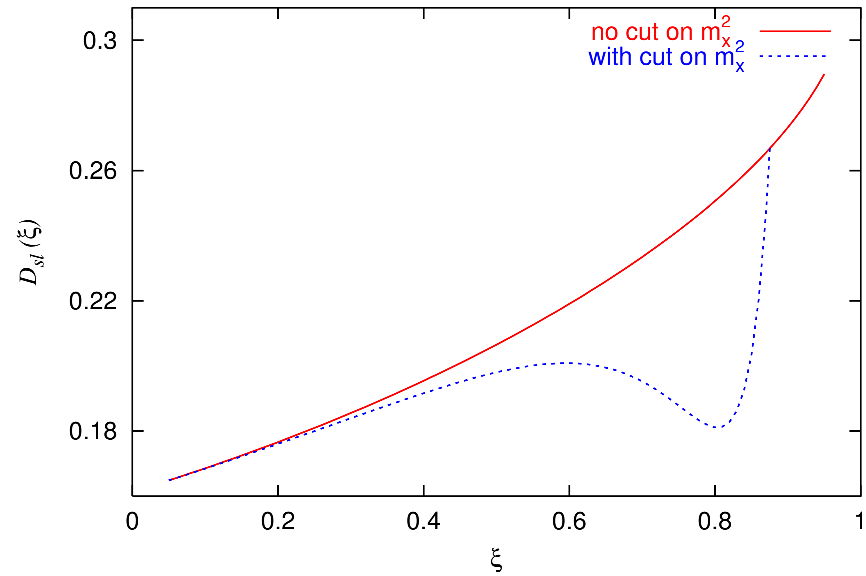
$$\boxed{\frac{d^3\Gamma_{sl,u}}{dx_l\,dw\,d\xi} = \left|V_{ub}\right|^2 \Gamma_{sl}^0 \left[K_{sl}(x_l, w; \alpha_s) f(\xi) + D_{sl}(x_l, w, \xi; \alpha_s)\right]}$$

$$\Gamma_{sl}^0 = \frac{G_F^2\, m_b^3\, \overline{m}_b^2}{192\,\pi^3}$$

- $f(\bullet)$ is the <u>*same*</u> function appearing in $B \to X_s \gamma$

Remainder functions





- relatively large contribution from remainder functions
- sizable theoretical uncertainty dominated by the μ dependence due to $\alpha_s(\mu)$

The "master formula"

$$\left.\frac{\dfrac{d\,BR_{sl,u}}{d\xi}-\left|\dfrac{V_{ub}}{V_{cb}}\right|^2\dfrac{BR_{sl,c}}{g_{sl}}D_{sl}(\xi;\alpha_s)}{\dfrac{d\,BR_{rd}}{dx}-\dfrac{6\alpha}{\pi}\left|\dfrac{V_{tb}V_{ts}^*}{V_{cb}}\right|^2\dfrac{BR_{sl,c}}{g_{sl}}D_{rd}(x;\alpha_s)}\right|_{x=\xi}=\frac{\pi}{6\alpha}\frac{K_{sl}(\alpha_s)}{K_{rd}(\alpha_s)}\left|\frac{V_{ub}}{V_{tb}V_{ts}^*}\right|^2$$

- equation involving *experimental* and *short-distance* quantities only
- l.h.s. is a function of $\xi = x$, r.h.s. is a constant
- correlation among $|V_{cb}|$, $|V_{ub}|$ and $|V_{tb}V_{ts}^*|$
- trivially extendible to partially-integrated *BR*s

Extracting $|V_{ub} / V_{cb}|$

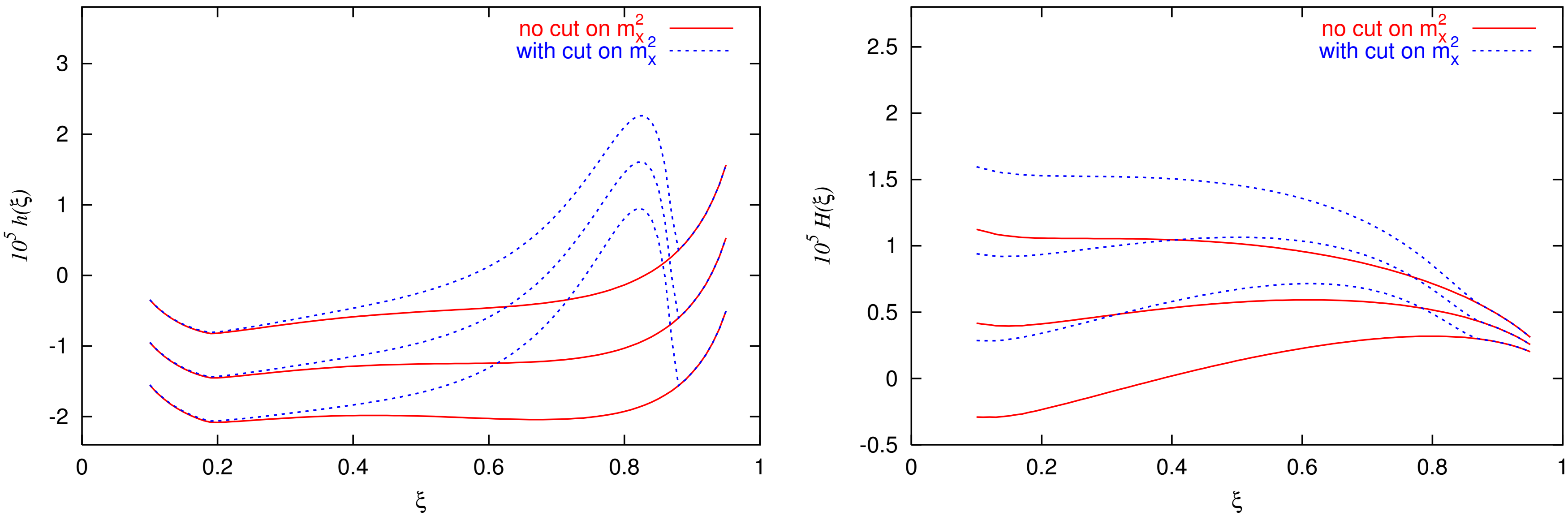
✔ In the approximation $|V_{cb}|^2 \sim |V_{tb} V_{ts}^*|^2$:

$$\left|\frac{V_{ub}}{V_{cb}}\right|^2 \simeq C(\alpha_s) \frac{\dfrac{d\,BR_{sl,u}}{d\xi}}{\left.\dfrac{dBR_{rd}}{dx}\right|_{x=\xi} - h(\xi;\alpha_s)}$$

$$C(\alpha_s) \equiv \frac{6\alpha}{\pi} \frac{K_{rd}(\alpha_s)}{K_{sl}(\alpha_s)} \simeq (1.94 \pm 0.16) \times 10^{-3}$$

$$h(\xi;\alpha_s) \equiv \frac{6\alpha}{\pi} \frac{BR_{sl,c}}{g_{sl}} \left[D_{rd}(\xi;\alpha_s) - \frac{K_{rd}(\alpha_s)}{K_{sl}(\alpha_s)} D_{sl}(\xi;\alpha_s) \right]$$

Cancellation of the remainder functions



$$h(\xi\,;\alpha_s) \sim 10^{-5}$$

- much smaller than the contribution of the spectra

Pros and Cons

+ only *measurable* and *NLO perturbative* quantities involved
+ well-defined IR subtraction scheme for the shape function
+ valid "point by point" or integrated over different ranges
+ no shape-function modeling nor specific moments needed
+ checks the theory of the shape function, higher-twist effects and the local quark-hadron duality
+ eventually allows extracting $|V_{ub}/V_{cb}|$ with a $O(5\%)$ theoretical error (if higher twists are negligible)

– higher twists may be non-negligible
– differences in the experimental resolution functions spoil the cancellation of the shape function: needs deconvolution